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Numerical treatment of a class of thermal contact problems

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Abstract

A mathematical problem modeling dynamical behavior of viscoelastic materials is studied. The problem is related to highly non-linear and non-smooth phenomena like contact, friction and a class of dynamic contact problems with friction and thermal effects. The applied approach leads to a system involving variational inequalities and a fully discrete scheme for numerical approximations and the analysis of error estimate order is provided. Numerical computations are given so as to illustrate the mathematical model.

Keywords: Viscoelastic materials; Sub-differential contact condition; Heat transfer; Friction; Numerical simulations

1. Introduction

Due to their growing complexity in the engineering sciences, contact problems for viscoelastic materials still remain a serious challenge. For example, contact phenomena are omnipresent in human skeletal systems, see for example [18]. Otherwise, many food materials used in process engineering are viscoelastic, see e.g. [11], and consequently, mathematical models can be very helpful in understanding various problems related to the product development, packing, transport, shelf life testing, thermal effects and heat transfer. It is thus important to study mathematical models that can be used to describe the dynamical behavior of a given viscoelastic material subjected to various highly nonlinear and even non-smooth phenomena like contact, friction and thermal effects.

The earlier studies relating to contact problems in the framework of variational formulation and infinitesimal deformations could be found in the reference works [6,12,19]. Thermoviscoelastic contact problems by taking into account the evolution of the temperature parameter could be found in [2,7,10]. Since considerable progress and development have been made in contact mechanics, as widely attested the pioneering works in [9,14,22,21], including mathematical modeling, mathematical analysis, with specific studies on the error estimates analysis, numerical approximation and numerical simulations. In these works the authors show the cross-fertilization between numerous frictional new models arising in contact mechanics and various types of abstract variational inequalities.

Further extensions to non convex contact conditions with non-monotone and possible multi-valued constitutive laws led to the active domain of non-smooth mechanics within the framework of the so-called hemivariational

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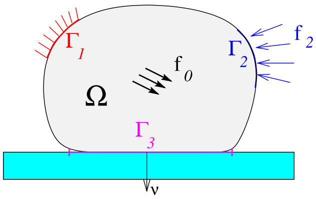
inequalities. For a mathematical as well as mechanical treatment we refer to [8,15,16,20]. A large deformation formulation for frictional contact problem has been treated in [5].

This work constitutes in some sense a continuation paper of the results obtained in [1]. The work in [1] has been devoted to a mathematical study of a class of dynamical long memory viscoelastic thermal problems where the contact is governed by a general sub-differential condition. Qualitative results like existence and uniqueness result of weak solutions on displacement and temperature fields have been proved but no numerical approximations have been performed.

Here we follow the latter work and propose a numerical scheme for the approximation of the solution fields so as to elaborate a general numerical analysis of error estimates. The theoretical results are then illustrated by different numerical simulations.

The paper is organized as follows. In Section 2 we give a short description of the mathematical model and recall the main existence and uniqueness result. In Section 3, we introduce a fully discrete approximation scheme, derive and prove an optimal order error estimate under certain solution regularity assumptions. Finally, in Section 4, we present a set of numerical simulations showing the evolution of the displacement field, the temperature field as well as the Von Mises stress norm.

2. The contact problems



The mechanical contact problem

A viscoelastic body occupies the domain Ω with surface Γ that is partitioned into three disjoint measurable parts Γ_1, Γ_2 and Γ_3, such that meas $(\Gamma_1) > 0$. Let $[0, T]$ be the time interval of interest, where $T > 0$. The body is clamped on $\Gamma_1 \times (0, T)$ and therefore the displacement field vanishes there. We also assume that a volume force of density $\boldsymbol{f}_0$ acts in $\Omega \times (0, T)$ and that surface traction of density $\boldsymbol{f}_2$ acts on $\Gamma_2 \times (0, T)$. The body may come in contact with an obstacle, the foundation, over the potential contact surface Γ_3. The model of the contact is specified by a general sub-differential boundary condition, where thermal effects may occur in the frictional contact with the basis. We are interested in the dynamic evolution of the body.

We denote by S_d the space of second order symmetric tensors on $\mathbb{R}^d$ $(d = 1, 2, 3)$ in practical case, while " $\cdot$ " and $|\cdot|$ will represent the inner product and the Euclidean norm on S_d and $\mathbb{R}^d$. Let $\Omega \subset \mathbb{R}^d$ be a bounded domain with a Lipschitz boundary Γ and let $\boldsymbol{\nu}$ denote the unit outer normal on Γ. Everywhere in the sequel the indexes i and j run from 1 to d, summation over repeated indices is implied and the index that follows a comma represents the partial derivative with respect to the corresponding component of the independent variable. We also use the following notation:

$$H = \left(L^2(\Omega)\right)^d, \qquad \mathcal{H} = \{\boldsymbol{\sigma} = (\sigma_{ij}) | \sigma_{ij} = \sigma_{ji} \in L^2(\Omega),\ 1 \le i, j \le d\},$$
$$H_1 = \{\boldsymbol{u} \in H | \boldsymbol{\varepsilon}(\boldsymbol{u}) \in \mathcal{H}\}, \qquad \mathcal{H}_1 = \{\boldsymbol{\sigma} \in \mathcal{H} | \text{Div}\,\boldsymbol{\sigma} \in H\}.$$

Here $\boldsymbol{\varepsilon} : H_1 \longrightarrow \mathcal{H}$ and Div $: \mathcal{H}_1 \longrightarrow H$ are the deformation and the divergence operators, respectively, defined by:

$$\boldsymbol{\varepsilon}(\boldsymbol{u}) = (\varepsilon_{ij}(\boldsymbol{u})), \quad \varepsilon_{ij}(\boldsymbol{u}) = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad \text{Div}\ \boldsymbol{\sigma} = (\sigma_{ij,j}).$$

The spaces H, $\mathcal{H}$, H_1 and $\mathcal{H}_1$ are real Hilbert spaces endowed with the canonical inner products given by:

$$(\boldsymbol{u}, \boldsymbol{v})_H = \int_\Omega u_i v_i\, dx, \qquad (\boldsymbol{\sigma}, \boldsymbol{\tau})_{\mathcal{H}} = \int_\Omega \sigma_{ij}\tau_{ij}\, dx,$$
$$(\boldsymbol{u}, \boldsymbol{v})_{H_1} = (\boldsymbol{u}, \boldsymbol{v})_H + (\boldsymbol{\varepsilon}(\boldsymbol{u}), \boldsymbol{\varepsilon}(\boldsymbol{v}))_{\mathcal{H}}, \qquad (\boldsymbol{\sigma}, \boldsymbol{\tau})_{\mathcal{H}_1} = (\boldsymbol{\sigma}, \boldsymbol{\tau})_{\mathcal{H}} + (\text{Div}\ \boldsymbol{\sigma}, \text{Div}\ \boldsymbol{\tau})_H.$$

We recall that C denotes the class of continuous functions and C^m, $m \in \mathbb{N}^*$ is the set of m times differentiable functions. Finally $\mathcal{D}(\Omega)$ denotes the set of infinitely differentiable real functions with compact support in Ω; and $W^{m,p}$,