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Rearranged nonlocal filters for signal denoising☆

G. Galiano*, J. Velasco

Dpto. de Matemáticas, Universidad de Oviedo, c/ Calvo Sotelo, 33007-Oviedo, Spain

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Abstract

In previous works, we investigated the use of local filters based on partial differential equations (PDE) to denoise one-dimensional signals through the image processing of time–frequency representations, such as the spectrogram. In these image denoising algorithms, the particularity of the image was hardly taken into account. We turn, in this paper, to study the performance of non-local filters, like Neighborhood or Yaroslavsky filters, in the same problem. The reformulation of the Neighborhood filter using the decreasing rearrangement allows us to implement an efficient algorithm. The integral histogram introduced by Porikli allows him in Porikli (2008) to obtain an implementation of the Yaroslavsky filter with a computational cost independent of the size of the box spatial local kernel. We heuristically justify the connection between the (fast) Neighborhood filter applied to a spectrogram and the corresponding Nonlocal Means filter (accurate) applied to the Wigner–Ville distribution of the signal. This correspondence holds only for time–frequency representations of one-dimensional signals, not to usual images, and in this sense the particularity of the image is exploited. We compare though a series of experiments on synthetic and biomedical signals the performance of local and non-local filters.

Keywords: Spectrogram; Image restoration; Local and non-local filtering; Instantaneous frequency; Biomedical signals

1. Introduction

Denoising one-dimensional signals is an important topic which is usually addressed from filter theory in time or frequency domains. In some applications in which processing speed is not a fundamental issue, filters defined in the joint time–frequency domain may be considered, usually improving the filtering process. Examples of this situation are found in Electrocardiogram (ECG) and other biomedical signals [24], human voice analysis [26] or animal sound analysis [12].

With respect to the latter, in previous works [6–8] we investigated the use of time–frequency distributions to estimate the number of wolves howling in a given recording to provide an estimation of the number of individuals in a pack. This estimation is the basis for counting regional wolf populations which is of interest for both ecological and economic purposes, since authorities must reimburse the cost of cattle killed by this protected species [22]. Of course,

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* Corresponding author. Tel.: +34 985103343; fax: +34 985103354.
E-mail addresses: galiano@uniovi.es (G. Galiano), julian@uniovi.es (J. Velasco).

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and despite the quality of recording devices, field recordings are usually affected for a variety of undesirable signals which range from low amplitude broad spectrum long duration signals, like wind, to signals localized in time, like cattle bells, or localized in spectrum, like car engines. Clearly, the addition of all these signals generates an unstructured noise in the background of the wolves chorus which must be treated for a proper signal analysis.

Medical signals are another good example of this situation. Due to the electromagnetic fields created by measuring devices, the usual low frequency signals to be acquired are contaminated by a background noise which is usually in the same frequency band as that of the signal of interest. Therefore, fine denoising techniques must be applied to segregate the signal of interest from the noise.

In general, the denoising procedure is not aimed to recovering a clean signal but to produce a clean time–frequency representation of the signal which allows further analysis techniques, for instance and importantly, the instantaneous frequency (IF) estimation. For the examples given above, IF estimation allows to count the number of different individuals howling in the recording (each individual being identified with an IF line). We also provide an example in which the spectrogram energy content of an ECG signal is filtered to identify an arrhythmia episode.

In [6,7,9], we used nonlinear diffusion image denoising techniques applied to the spectrogram of a sound signal, a wolf chorus. Although, as mentioned above, execution time is not a relevant issue for this type of problems, we found that nonlinear diffusion algorithms require a high computational time, making their use not operative in many situations. In addition, these filters do not take advantage of the special characteristics of the image produced from the spectrogram, i.e. they operate on the spectrogram as in any other image. In this article we show that nonlocal filters such as Neighborhood filters [13] are computationally more efficient to deal with these images and give similar results. Moreover, we point out a relationship between the Nonlocal Means filter [3] and the Neighborhood filter which is exclusive of their implementation on images defined through time–frequency distributions.

The outline of the article is as follows. We present in Section 2 the mathematical framework of the problem and the filtering techniques proposed in this article for one-dimensional signal denoising. In particular, we justify our choice of the Neighborhood filter as an inexpensive approximation to the well known Nonlocal Means filter for the special case of spectrogram images. In Section 3, we introduce the discrete problem and deduce the corresponding formulas for algorithm implementation. Apart from the Neighborhood filter, we consider the Yaroslavsky–SUSAN [27,23] filter and a nonlinear diffusion filter based in the Total Variation norm [20,9], for comparison purposes. Then, we demonstrate the performance of these filters by applying them to three noisy signals (synthetic, wolf chorus and ECG) and give quantitative comparisons based on the Mean Square Error (MSE), and the visual inspection of the processed spectrograms and other related magnitudes.

2. Mathematical framework

Let $f \in L^2(\mathbb{R})$ denote a one-dimensional signal and $\mathrm{WV}(f;\cdot,\cdot)$ be its Wigner–Ville distribution, defined as

$$\mathrm{WV}(f;t,\omega) = \int_{\mathbb{R}} f\left(t+\frac{s}{2}\right)\bar{f}\left(t-\frac{s}{2}\right)e^{-iws}ds,$$

where $\bar{f}$ denotes the complex conjugate of f. The Wigner–Ville distribution has received much attention for IF estimation due to its excellent concentration for mono-signals and many other desirable mathematical properties, see [16]. However, it is well known that it presents high amplitude sign-varying cross-terms for multi-component signals which make its interpretation difficult. For attenuating these interference terms several approaches have been followed, mainly based on the smoothing of the WV by convolution with a suitable regularizing kernel. Special mention is due to the spectrogram, which may be defined either as the energy density function of the short time Fourier transform

$$\mathcal{G}_\varphi(f;t,\omega) = \int_{\mathbb{R}} f(s)\varphi(s-t)e^{-i\omega s}ds, \tag{1}$$

for some real, symmetric and normalized window $\varphi \in L^2(\mathbb{R})$, i.e.

$$S_\varphi(f;t,\omega) = \mathcal{G}_\varphi(f;t,\omega)\bar{\mathcal{G}}_\varphi(f;t,\omega), \tag{2}$$

or as the convolution product of the Wigner–Ville distributions of the signal and the window

$$S_\varphi(f;t,\omega) = \int_{\mathbb{R}^2} \mathrm{WV}(\varphi;\tilde{t},\tilde{\omega})\mathrm{WV}(f;t-\tilde{t},\omega-\tilde{\omega})d\tilde{t}d\tilde{\omega}. \tag{3}$$