Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 118 (2015) 343–359
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

Trivariate spline quasi-interpolants based on simplex splines and polar forms☆

A. Serghini*, A. Tijini

University Mohammed I, ESTO, MATSI Laboratory, Oujda, Morocco

Received 4 March 2014; received in revised form 10 November 2014; accepted 21 November 2014
Available online 29 November 2014

Abstract

In this work we describe an approximating scheme based on simplex splines on a tetrahedral partition using volumetric data. We use the trivariate simplex B-spline representation of polynomials or piecewise polynomials in terms of their polar forms to construct several differential or discrete quasi interpolants which have an optimal approximation order.
© 2014 International Association for Mathematics and Computers in Simulation (IMACS). Published by Elsevier B.V. All rights reserved.

Keywords: Polar form; Quasi-interpolation; Simplex B-spline

1. Introduction

Multivariate simplex splines for approximation theory have been extensively investigated in mathematical science for many years. Motivated by an idea of Schoenberg for a geometric interpretation of univariate B-splines, de Boor [8] first presented a brief description of multivariate simplex splines. Since then, their theoretical aspects have been explored extensively. From the point of view of blossoming (see [12,13,27], for instance), Dahmen et al. [7] proposed triangular B-splines called also DMS-splines. Greiner and Seidel [14] demonstrated the practical feasibility of multivariate B-splines algorithms in graphics and shape design. Pfeifle and Seidel [25,26] proposed a fast evaluation technique for a quadratic bivariate DMS-splines surface and demonstrated the fitting of triangular B-spline surfaces to scattered data through the use of least squares and optimization techniques. Other bivariate and trivariate spline spaces are studied and analyzed in [16].

In recent years, the reconstruction of volumetric data became an active area of research since it is important for many applications such as scientific visualization, medical imaging and solid modeling. To model volumetric objects with high-order continuity, techniques based on splines are frequently used. Nonetheless, modeling with B-splines has several shortcomings. Its modeling scope is extremely constrained in terms of geometric and topological aspects. The several existing studies (see [1,20,22,24], for instance) use the tensor product splines in three variables. Standard examples are trilinear, continuous splines, where the total degree of the polynomial pieces is three, as well as triquadratic

☆ Research supported by URAC-05.
* Corresponding author.
E-mail addresses: a.serghini@ump.ma (A. Serghini), tijiniahmed@yahoo.fr (A. Tijini).

http://dx.doi.org/10.1016/j.matcom.2014.11.008
0378-4754/© 2014 International Association for Mathematics and Computers in Simulation (IMACS). Published by Elsevier B.V. All rights reserved.

and tricubic, smooth splines, where the total degree of the polynomial pieces are six and nine, respectively. A local and smooth model of the reconstruction of volume data, close to tensor-product schemes, is represented by blending sums of univariate and bivariate C^1 quadratic spline quasi-interpolants in [29–31].

To overcome the difficulties due to the trivariate tensor product splines, Nürnberger et al. [23] and T. Sorokina et al. [35] constructed on type-6 tetrahedral partition a trivariate quadratic and cubic piecewise polynomials quasi-interpolants which are based on trivariate splines of lowest possible degree. In these methods, the Bernstein–Bézier coefficients of the spline quasi-interpolants are immediately available from the given values by applying local averaging. Despite the elegance of these methods, the obtained quasi-interpolants do not reproduce the whole space $\mathbb{P}_2(\mathbb{R}^3)$ of trivariate polynomials of degree less than or equal to 2, and they are only second order accurate. In [28], S. Remogna constructed new quasi-interpolation schemes based on the trivariate C^2 quartic box spline defined on a type-6 tetrahedral partition with a fourth approximation order.

Spline quasi-interpolants are very useful approximants in practice. In general, a spline quasi-interpolant for a given function f is obtained as linear combination of some elements of a suitable set of basis functions, i.e;

$$\mathcal{Q}f = \sum_{j\in\mathcal{J}} \mu_j(f)B_j,$$

where $\{B_j, j \in \mathcal{J}\}$ is the B-spline basis of some spline space. If $\mu_j(f)$ is a linear combination of values of derivatives of f, the associated quasi-interpolant is called a *differential quasi-interpolant* (abbr. DQI) and if $\mu_j(f)$ is a linear combination of discrete values of f, the associated quasi-interpolant is called a *discrete quasi-interpolant* (abbr. dQI) (see [30], for instance). In this paper, we use the trivariate simplex B-splines which are piecewise polynomials of the lowest possible degree and the highest possible smoothness everywhere for constructing discrete and differential simplex spline quasi-interpolants which have an optimal approximation order. We extend the approach given in [32,33] to $\mathbb{R}^3$ and to simplex splines of any degree. It is based on the polar form of a chosen local polynomial approximant like a local interpolant or another approximant having the optimal approximation order.

The paper is organized as follows. In Section 2 we give some definitions and properties of trivariate simplex B-splines. In Section 3 we introduce the B-spline representation of all trivariate polynomials or DMS-splines over a tetrahedral partition Δ of a bounded domain $D \subset \mathbb{R}^3$, in terms of their polar forms. In Section 4 we apply the approach introduced in [32] to trivariate DMS B-splines. Then we describe some differential and discrete simplex spline quasi interpolants in three variables which reproduce trivariate polynomials and provide the full approximation order in the space of trivariate simplex splines. Some interesting results concerning the quadratic case are developed in Section 5. In Section 6 we give some upper bounds of the infinity norms of some families of trivariate discrete quasi-interpolants. Finally, some numerical examples are proposed in Section 7.

2. Trivariate DMS-splines

In this subsection, we review some basic definitions and properties of trivariate simplex splines. The following results can be found in [2,3,5–7,14,15,21,26,34].

For any ordered set of affinely independent points $W = \{w_0, w_1, w_2, w_3\}$ of $\mathbb{R}^3$, we define

$$d(W) = \det\begin{pmatrix} 1 & 1 & 1 & 1 \\ w_0 & w_1 & w_2 & w_3 \end{pmatrix}.$$

Let $V = \{v_0, \ldots, v_m\}$ be an arbitrary set in $\mathbb{R}^3$, we denote by $[V] = [v_0, \ldots, v_m]$ the convex hull of the set V. if we denote by e_i, $i = 1, 2, 3$ the unit vectors of $\mathbb{R}^3$, the half-open convex hull of V is defined as

$$[V) = \{x \in [V]/\exists\epsilon_1, \epsilon_2, \epsilon_3, \forall 0 \le \alpha_1 \le \alpha_2 \le \alpha_3 \le 1,\ (x + \alpha_1\epsilon_1 e_1 + \alpha_2\epsilon_2 e_2 + \alpha_3\epsilon_3 e_3) \in [V]\},$$

i.e., x belongs to the half open convex hull $[V)$ of V, if there exists a small tetrahedron that lies entirely within the convex hull $[V]$.

The simplex B-spline $M(x|V) = M(x|v_0, \ldots, v_m)$ is defined recursively as follows: When $m = 3$ we set

$$M(x|V) = \frac{\mathcal{X}_{[v_0,v_1,v_2,v_3)}(x)}{|d(V)|}, \tag{2.1}$$

Download English Version:

https://daneshyari.com/en/article/1139320

Download Persian Version:

https://daneshyari.com/article/1139320

Daneshyari.com