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Original article

Numerical simulations of traveling wave solutions in a drift paradox inspired diffusive delay population model

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Abstract

We describe numerical algorithm for the simulation of traveling wave solutions in a newly formulated drift paradox inspired diffusive delay population model. We use method of lines to discretize the boundary value problem for the reaction-diffusion equations and we integrate in time the resulting system of delay differential equations using the embedded pair of continuous Runge–Kutta methods of order four and three. We advance the solution with the method of order four and the approximations of order three are used for local error estimation. Numerical results demonstrate the robustness, efficiency, and accuracy of our approach. Moreover, these numerical results confirm the recent theoretical results on the minimum traveling wave speed for this model.

Keywords: Reaction-diffusion equations; Traveling wave solutions; Drift paradox; Numerical simulations; Delay models

1. Introduction

Many organisms persist in their environments despite the presence of constant advection into often unfavorable habitats. Such intriguing phenomena is referred as the “drift paradox” in ecological literature. Examples include plants with windborn seeds, organisms in rivers and estuaries, and marine organisms with larval dispersal influenced by ocean currents [1]. A closer look at such species growth and disperse activities reveals that various specific biological and physical processes can contribute to the persistence of a given organism in such an environment [5,6].

There is a significant and growing interest in modeling population growths in a setting mixing advection with diffusion [2,3,10,17]. For example, the paper by Pachepsky et al. [16] studied such a population growth with additional assumptions that the reproduction occurs only in the stationary phase and the population can be divided into two interacting compartments: individuals residing on the benthos and individuals drifting in the flow. They proved that persistence of the population is guaranteed if at low population densities the local growth rate of the stationary component of the population exceeds the rate of entry of individuals into the drift. In [11], the authors incorporated a

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maturation delay τ in the model of Pachepsky et al. [16] which yielded the following two-dimensional single-species diffusive-delay population model

$$\begin{cases} \dfrac{\partial n_d}{\partial t}(x,t) = \delta\, n_b(x,t) - \sigma\, n_d(x,t) - v\,\dfrac{\partial n_d}{\partial x}(x,t) + d\,\dfrac{\partial^2 n_d}{\partial x^2}(x,t), \\ \dfrac{\partial n_b}{\partial t}(x,t) = r\, n_b(x,t-\tau) - \dfrac{r}{\kappa} n_b^2(x,t) - \delta\, n_b(x,t) + \sigma\, n_d(x,t), \end{cases} \tag{1.1}$$

$x \in [a, b]$, $t \geq 0$, with initial and boundary conditions

$$\begin{cases} n_d(x,0) = n_d^0(x), \quad n_b(x,t) = n_b^0(x,t), \quad x \in [a,b], \quad t \in [-\tau, 0], \\ n_d(a,t) = g_1(t), \quad n_d(b,t) = g_2(t), \quad t \geq 0. \end{cases} \tag{1.2}$$

Here, $n_b(x, t)$ is the population density of the benthos, $n_d(x, t)$ is the population density of the drift, the delay τ is the time taken from birth to maturity, r is the rate of the benthic population at which individuals are born, κ is the carrying capacity, δ is the per capita rate at which individuals in the benthic population enter the drift, σ is the per capita rate at which the organism return to benthic population from drifting, d is the diffusion coefficient, v is the advection speed experienced by the organisms, and $n_d^0(x)$, $n_b^0(x, t)$, $g_1(t)$, and $g_2(t)$ are given functions. Inheriting the assumptions proposed in [16], the diffusion term d for the drift compartment represents the effect of heterogeneous stream flows and random individual swimming.

Model (1.1) is a cooperative delayed reaction-diffusion system. The existence of analytic solution in (1.1) with appropriate initial and boundary conditions is ensured by the results in Martin and Smith [12]. It was established in [11] that the model (1.1) admits traveling wave solutions. In this paper we propose a numerical algorithm for the numerical simulations of these solutions. This method can be easily adapted to simulate other diffusive delay models such as those studied by Gourley and Kuang [8,9].

2. Discretization in space

Let be given an integer $N > 0$ and consider the uniform grid $x_i = a + ih$, $i = 0, 1, \ldots, N+1$, where $(N+1)h = b - a$. Putting $x = x_i$, $i = 1, 2 \ldots, N$, into (1.1) and discretizing the partial derivative $\partial n_d/\partial x$ at the points (x_i, t) by the central differences

$$\frac{\partial n_d}{\partial x}(x_i,t) \approx \frac{u_d(x_{i+1},t) - u_d(x_{i-1},t)}{2h},$$

and the partial derivative $\partial^2 n_d/\partial x^2$ at the points (x_i, t) by the finite differences of second order

$$\frac{\partial^2 n_d}{\partial x^2}(x_i,t) \approx \frac{n_d(x_{i+1},t) - 2\,u_d(x_i,t) + u_d(x_{i-1},t)}{h^2}$$

we obtain a system of the delay differential equations of the form

$$\begin{cases} n_{d,i}'(t) = \delta\, n_{b,i}(t) - \sigma\, n_{d,i}(t) - v\,\dfrac{n_{d,i+1}(t) - n_{d,i-1}(t)}{2h} \\ \qquad + d\,\dfrac{n_{d,i+1}(t) - 2\,u_{d,i}(t) + n_{d,i-1}(t)}{h^2}, \\ n_{b,i}'(t) = r\, n_{b,i}(t-\tau) - \dfrac{r}{\kappa} n_{b,i}^2(t) - \delta\, n_{b,i}(t) + \sigma\, n_{d,i}(t), \end{cases} \tag{2.1}$$

$i = 1, 2, \ldots, N$. Here, $n_{d,i}(t)$ are approximations to $n_d(x_i, t)$ and $n_{b,i}(t)$, $n_{b,i}(t-\tau)$ are approximations to $n_b(x_i, t)$, $n_b(x_i, t-\tau)$, respectively. For $i = 1$ and $i = N$ we have to incorporate into the above equations the boundary conditions from (1.2), i.e.,

$$n_{d,0}(t) \approx n_d(a,t) = g_1(t) \quad \text{and} \quad n_{d,N+1}(t) \approx n_d(b,t) = g_2(t).$$