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On the dynamics of economic games based on product differentiation

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Abstract

Defining a micro-economics-based demand structure of differentiated products, we analyse the conditions for stability of the Cournot–Nash and the Bertrand–Nash equilibria. We find that, in the presence of both complements and substitute goods, the stability of the Nash equilibrium increases when goods tend to be independent. Moreover, the Nash equilibrium is also more stable under quantity competition than under price competition, regardless of whether goods are complements or substitutes.

Keywords: Product differentiation; Bertrand–Nash competition; Cournot–Nash competition; Bounded rationality; Dynamic stability

1. Introduction

The predictions of oligopoly theory crucially depend on behavioural assumptions about how a firm conjectures other firms will react to its own actions. Cournot [6] made the assumption that firms are naive players. Thus, each firm expects the rival to offer the same quantity in the current period as in the previous period, and there is no retaliation at all.

Later contributions [5,8] varied this assumption and proposed alternative solutions, initiating the conjectural variations literature. However, as Friedman and Mezzetti [7] note, the conjectural variations model is static, and describing a firm's beliefs about opponents' reactions to its own choice requires a dynamic setup.

Therefore, different approaches to firm behaviour, and more realistic rules of expectations have been proposed in a dynamic context. This is the case with the notion of bounded rationality [4,1], and the adaptive expectations principle [3]. Several works have shown that the Cournot model may lead to periodic cycles and deterministic chaos.

Recently, Askar and Alshamrani [2] analyse the stability of the Nash equilibrium under both Cournot and Bertrand competition, with three competitors, in a model of horizontal product differentiation. They conclude that the stability/instability of the Nash equilibrium is sensitive to the game parameters.

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The present paper constitutes a revision of the above article. In contrast to the cited work [2], we introduce a micro-economics-based demand structure of differentiated products, which may be substitutes or complements for each other. In this setting, the conditions for the stability of the Cournot–Nash equilibrium and the Bertrand–Nash equilibrium are correctly studied, showing that the conclusions offered in [2] are not reproduced.

We find that under both quantity and price competition, the Nash equilibrium is more stable when goods are more independent. Moreover, in the presence of both complements and substitutes, the Cournot–Nash equilibrium is more stable than the Bertrand–Nash equilibrium.

The remainder of the paper is organized as follows. The revised model of [2] is presented in Section 2. Section 3 analyses the stability of the Bertrand–Nash equilibrium. Section 4 develops the stability of the Nash equilibrium under Cournot competition. The comparison between both scenarios is analysed in Section 5. To show the obtained results and the behaviour of the solution trajectories of the dynamic systems, Section 6 includes numerical simulations. Finally, Section 7 closes the paper with a review of our main conclusions.

2. The model

Following [2], we develop a generalized version of the model formulated by Singh and Vives [10] for the triopoly case. Specifically, we consider an economy with a monopolistic sector of three firms, each producing a differentiated good, and a competitive numeraire sector. There is a continuum of identical consumers with a utility function separable and linear in the numeraire good.

With q_i being the amount of good i, and p_i its price, the representative consumer maximizes the utility function with respect to the quantities:

$$V(q_1, q_2, q_3) = u(q_1, q_2, q_3) - \sum_{i=1}^{3} p_i q_i. \tag{1}$$

Function $u(q_1, q_2, q_3)$ is assumed to be quadratic and must be strictly concave.[2] Particularly, we adopt the following specification:

$$u(q_1, q_2, q_3) = \alpha_1 q_1 + \alpha_2 q_2 + \alpha_3 q_3 - \frac{\beta_1 q_1^2 + \beta_2 q_2^2 + \beta_3 q_3^2 + 2\gamma_{12} q_1 q_2 + 2\gamma_{13} q_1 q_3 + 2\gamma_{23} q_2 q_3}{2}. \tag{2}$$

For the sake of simplicity, we assume that $\alpha_1 = \alpha_2 = \alpha_3 = a > 0$, $\beta_1 = \beta_2 = \beta_3 = 1 > 0$, and $\gamma_{12} = \gamma_{13} = \gamma_{23} = d$.

In order to ensure the concavity of (2) we impose $-\frac{1}{2} < d < 1$.

Maximizing (1), given (2), we deduce a linear demand structure, which is the inverse demand in the region of quantity where prices are positive:

$$\left.\begin{array}{l} p_1 = a - q_1 - dq_2 - dq_3 \\ p_2 = a - q_2 - dq_1 - dq_3 \\ p_3 = a - q_3 - dq_1 - dq_2 \end{array}\right\}. \tag{3}$$

From (3), we can obtain the direct demands as:

$$\left.\begin{array}{l} q_1 = \dfrac{a}{1+2d} - \dfrac{(1+d)p_1}{(1-d)(1+2d)} + \dfrac{dp_2}{(1-d)(1+2d)} + \dfrac{dp_3}{(1-d)(1+2d)} \\ q_2 = \dfrac{a}{1+2d} - \dfrac{(1+d)p_2}{(1-d)(1+2d)} + \dfrac{dp_1}{(1-d)(1+2d)} + \dfrac{dp_3}{(1-d)(1+2d)} \\ q_3 = \dfrac{a}{1+2d} - \dfrac{(1+d)p_3}{(1-d)(1+2d)} + \dfrac{dp_1}{(1-d)(1+2d)} + \dfrac{dp_2}{(1-d)(1+2d)} \end{array}\right\}. \tag{4}$$

From (4), it is deduced that, when $d = 0$, goods are independent and each firm is monopolist in the market for its variety. If $-\frac{1}{2} < d < 0$, goods are complements, and if instead, $0 < d < 1$, goods are substitutes, being perfect substitutes for the limit case $d = 1$.

[2] The utility function proposed by Askar and Alshamrani [2] is not concave. Therefore, the existence and unicity of the consumer equilibrium is not guaranteed, and the associated demand system is not well-defined.