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Original article

Attracting and invariant sets of non-autonomous reaction-diffusion neural networks with time-varying delays

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Abstract

In this paper, a class of non-autonomous reaction-diffusion neural networks with time-varying delays is investigated. By establishing a new differential inequality and employing the properties of spectral radius of nonnegative matrix and diffusion operator, the global attracting and positive invariant sets and exponential stability of non-autonomous reaction-diffusion neural networks with time-varying delays are obtained. Our results do not require the conditions of boundedness of the coefficient of neural networks. One example is given to illustrate the effectiveness of our conclusion.

Keywords: Attracting and invariant sets; Non-autonomous; Reaction-diffusion neural networks; Differential inequality; Delays

1. Introduction

Recently, various neural network models have been extensively investigated and successfully applied to image processing, signal processing, pattern classification, optimization problem, etc. In implementation of neural networks, time delays are unavoidable due to finite switching speeds of the amplifiers and communications. The existence of time delays may destroy a stable network and cause sustained oscillations, bifurcation or chaos and thus could be harmful. Therefore, the stability of neural networks with delay have been studied widely (see Refs. [6,9,11,12,15,22,21,27,29]). However, strictly speaking, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields. For example, as it is well known, the multi-layer cellular neural networks are arrays of nonlinear and simple computing elements characterized by local interactions between cells, therefore the multi-layer cellular neural networks paradigm are well suited to describe locally interconnected simple dynamical systems showing a lattice-like structure (see Refs. [2,3]). In other words, the whole structure and dynamic behavior of multi-layer cellular neural networks are seriously dependent on the evolution time of each variable and its position (space), but also intensively dependent on its interactions deriving from the space-distributed structure of the whole networks. So, we must consider that the activations vary in space as well as in time. In recent years, various interesting results on the stability and other behaviors of autonomous neural networks with reaction-diffusion terms have been reported (see Refs. [14,18,19,23,31,32]).

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It is well known that the non-autonomous phenomenon often occurs in many realistic systems. Particularly, when we consider a long-term dynamical behaviors of a system, the parameters of the system are usually subjected to environmental disturbances and frequently vary with time. In this case, non-autonomous neural network model can even accurately depict evolutionary processes of networks. Thus the research on the non-autonomous neural networks is also important like on autonomous neural networks (see Refs. [7,13,16,17,20,25,26,28,30]). In [13,16,17,25,30], authors have considered the boundedness or exponential stability for non-autonomous neural networks with delays and reaction-diffusion terms, but the boundedness of coefficient is required, this limited the application of the neural networks in some extent. In addition to the stability of neural networks, the attracting and invariant sets of neural networks are also important (see Refs. [4,7,8,28]). In [28], authors have considered the attracting and invariant sets of non-autonomous neural networks with reaction-diffusion terms, but the common factors of coefficient are required. Motivated by this, the main purpose of this paper is to study the attracting and invariant sets of non-autonomous reaction-diffusion neural networks with time-varying delays. In this paper, we first establish a new differential-inequality to improve the inequalities established in [10,33]. Basing on the new differential-inequality and employing the properties of spectral radius of nonnegative matrix and diffusion operator, without requiring the boundedness and common factors of coefficient of neural networks, the global attracting and positive invariant sets and the sufficient conditions ensuring the exponential stability of non-autonomous reaction-diffusion neural networks with time-varying delays are obtained. Our results can generalize and improve the existing works.

2. Model description and preliminaries

In this paper, we consider the following non-autonomous reaction-diffusion neural networks with time-varying delays

$$\begin{cases} \dfrac{\partial u_i(t,x)}{\partial t} = \displaystyle\sum_{k=1}^{m} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right) - a_i(t)u_i(t,x) \\ \qquad\qquad + \displaystyle\sum_{j=1}^{n} b_{ij}(t)f_j(u_j(t,x)) + \sum_{j=1}^{n} c_{ij}(t)g_j(u_j(t-\tau_{ij}(t),x)) + I_i(t) \\ \dfrac{\partial u_i(t,x)}{\partial n} := \left(\dfrac{\partial u_i(t,x)}{\partial x_1}, \ldots, \dfrac{\partial u_i(t,x)}{\partial x_m}\right)^T = 0, \quad t \geq t_0 \geq 0,\ x \in \partial\Omega, \\ u_i(t_0+s,x) = \phi_i(s,x), \quad -\tau \leq s \leq 0,\ x \in \Omega, \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, n$, and n corresponds to the number of units in a neural network; $\Omega \subset R^m$ is a bounded domain with smooth boundary $\partial\Omega$ and measure $\mu = \text{mes}\Omega > 0$; x_k $(k = 1, 2, \ldots, m)$ corresponds to the kth coordinate in the space; $u_i(t, x)$ is the state of the ith unit at time t and in space x; $f_j(\cdot)$ and $g_j(\cdot)$ denote the activation functions of the jth unit at time t and in space x; $\tau_{ij}(t)$ is the transmission delay at time t with $0 \leq \tau_{ij}(t) \leq \tau$ and $\tau > 0$ is a constant; smooth function $D_{ik} = D_{ik}(t, x) \geq 0$ corresponds to the transmission diffusion operator along the ith unit; $a_i(t) > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $b_{ij}(t)$, $c_{ij}(t)$ denote the strength of the jth neuron on ith unit at time t and $t - \tau_{ij}(t)$, respectively; $I_i(t)$ denote the bias of the ith unit at time t; $\phi(s, x) = (\phi_1(s, x), \phi_2(s, x), \ldots, \phi_n(s, x))^T$ is the initial value.

Next, we introduce several notations and recall some basic definitions.

Let R^n be the space of n-dimensional real column vectors, $\mathcal{N} \triangleq \{1, 2, \ldots, n\}$, $R_+ \triangleq [0, +\infty)$, and $R^{m\times n}$ denote the set of $m \times n$ real matrices. Usually E denotes an $n \times n$ unit matrix. For $A, B \in R^{m\times n}$ or $A, B \in R^n$, the notation $A \geq B$ $(A > B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$\geq(>)$". Especially, $A \in R^{m\times n}$ is called a nonnegative matrix if $A \geq 0$, and $z \in R^n$ is called a positive vector if $z > 0$. Let $\rho(A)$ denote the spectral radius of square matrix A.