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Generalized chaos control and synchronization by nonlinear high-order approach

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Abstract

This paper investigates the generalized control and synchronization of chaotic dynamical systems. First, we show that it is possible to stabilize the unstable periodic orbits (UPOs) when we use a high-order derivation of the OGY control that is known as one of useful methods for controlling chaotic systems. Then we examine synchronization of identical chaotic systems coupled in a master/slave manner. A rigorous criterion based on the transverse stability is presented which, if satisfied, guarantees that synchronization is asymptotically stable. The Rössler attractor and Chen system are used as examples to demonstrate the effectiveness of the developed approach and the improvement over some existing results.

Keywords: Chaotic systems; High-order control; Synchronization; Transverse stability

1. Introduction

Nowadays, chaos control and synchronization are important topics in the nonlinear control systems. Chaos control can be understood as the use of small perturbations to stabilize unstable periodic orbits (UPOs) embedded in chaotic systems via small control input. This concept was first initiated by Ott, Grebogi and Yorke known as OGY method [31]. However, the OGY method requires exact calculation of the UPO, which is often very hard in experiment. An alternative control method was proposed by Pyragas which states that, chaotic system can be stabilized by a feedback perturbation proportional to the difference between the present and the delayed state of the system [34]. However, it has been shown that the Pyragas method also has a limitation related to the odd number property [28,44]. Numerous research efforts are dedicated to overcome some limitations of these original methods. In fact, some improvements concerning the OGY method are reported in [2,7,12,19,35]. Based on the Pyragas method, several methods avoiding the odd number property are given in [1,17,20]. Boccaletti et al. [5] give a survey of the most relevant control methods. Recent progresses in controlling chaos can be found in [41,42]. On the other hand, since Fujisaka and Yamada's 1983 paper on synchronized motion in coupled chaotic systems [13], many researchers have discussed the stability of this type of motion. Up to now, different methods and techniques are investigated on synchronization of chaotic systems. Here, we just mention the Pecora–Carroll synchronization [33], complete synchronization [47], phase and lag synchronizations [36,37], generalized synchronization [18,22,46], partial synchronization [49], predictive synchronization [39], and

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adaptive–impulsive synchronization [45]. Boccaletti et al. [6] and Nijmeijer [29] give an overview of the proposed synchronization methods. Banerjee [3] and Stavroulakis [43] derived important results in secure communication using chaos synchronization. The discussion in this paper will center around the type of synchronization discussed by most of these authors. Namely, two or more identical chaotic systems, coupled in a master/slave manner, which exhibit motion that is chaotic and identical in time. Another interesting topic concerns the stability of the synchronized state to a mismatch of the parameter values between the two chaotic systems. Recent studies of this problem has led to the observation of new phenomena, such as riddled basins of attraction, attractor bubbling, on of intermittency and blow-out bifurcation [23,26,32,48].

The purpose of this paper is to generalize the high-order chaos control approach [7,8], and then to study the synchronization process of coupled controlled chaotic systems. First, the Poincaré section is employed to identify unstable periodic orbits embedded in the chaotic systems. Since a generic unstable periodic orbit is mapped on the Poincaré section by an ordered sequence of crossing points, the controller parameters are determined for the desired unstable periodic orbit evaluating the influence of small parametric variation on the unstable periodic orbit variation. Afterward, the transverse stability criterion is employed to synchronize identical chaotic attractors. We continue the study by examining the bifurcations through which low periodic orbits embedded in the synchronized chaotic state lose their transverse stability and produce the characteristic phenomena of local and global riddling, blow-out bifurcations, attractor bubbling, on–off intermittency, etc. As a potential application of the proposed control and synchronization strategy, we used it to study the control of unstable periodic orbits, and synchronization of coupled Rössler and Chen chaotic dynamical systems.

This paper is organized as follows. After this introduction, we give in Section 2 the methodology of generalized control and synchronization by nonlinear high-order approach. In Section 3, we discuss chaos control and synchronization of two identical Rössler and two identical Chen dynamical systems, and numerical simulations are given to show this process. Conclusion is given in Section 4.

2. Nonlinear high-order control and synchronization principles

Consider the two nonlinear systems modeled by a set of ordinary nonlinear differential equations as follows:

$$\dot{X}_1(t) = f(X_1(t), \alpha) \tag{1}$$

$$\dot{X}_2(t) = g(X_1(t), X_2(t)) \tag{2}$$

where $f, g : R^N \to R^N$ are continuous functions, $X_1, X_2 \in R^N$ are the state variables and $\alpha \in R$ is a control parameter.

The system given by Eq. (1) will be called the master system and the system given by Eq. (2) will be called the slave system.

2.1. Control principle

The chaos control method may be understood as a two-stage technique. In the first step, the learning stage, the unstable periodic orbits are identified and control parameters are evaluated. After that, there is the control stage where the desired unstable periodic orbit is stabilized.

The learning stage uses, in a large sense, the Poincaré section (PS) properties. For this, we determine the influence of control parameter on the original unstable periodic orbit. Secondly, we determine the variation that should be applied to the control parameter in order to force the system to rejoin the desired unstable periodic orbit. After information about the PS has been gathered, the system is kept to remain on the desired orbit by perturbing the appropriate parameter. Similar to the original OGY control method, we wish to make only small controlling perturbations to the system. We do not envision creating new orbits with different properties from the already existing orbits.

Let us consider the chaotic system (1), where x is a N dimensional vector and α a control parameter, and start by setting α in order to have a stable periodic orbit of period τ. A $N-1$ dimensional PS intercepts the orbit at a point which repeats after a time τ. Let $\overline{X}$ be an unstable periodic orbit of the chaotic system (1) represented by a point at PS.