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Mathematics and Computers in Simulation 94 (2013) 1–14
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original article

The dynamical complexity of a predator–prey system with Hassell–Varley functional response and impulsive effect☆

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Received 23 June 2012; received in revised form 15 January 2013; accepted 20 May 2013
Available online 14 June 2013

Abstract

In this paper, the dynamics of an impulsively controlled predator–prey system with the Hassell–Varley functional response are studied. Under impulsive control, the conditions for the existence of a stable prey-free solution and for the permanence of the system are investigated by using Floquet theory and comparison theorems. Also the existence of a nontrivial periodic solution under some conditions is shown via the bifurcation theorem. Finally, numerical simulations are given to substantiate our theoretical results and to illustrate various dynamical behaviors of the system.

MSC: 34A37; 34D23; 34H05; 92D25

Keywords: Predator–prey system; Hassell–Varleyfunctional response; Impulsive differential equation; Floquetthoery; Bifurcation theorem

1. Introduction

One of the important components of the predator–prey relationships is the predator's rate of feeding on prey, i.e., the so-called predator's functional response[1,5,16]. It refers to the change in the density of prey attached per unit time per predator as the prey density changes. Functional response equations that are strictly prey-dependent, such as the Holling Tanner types, are predominant in the literature[9,17]. However, the prey-dependent functional responses fail to system the interference among predators. Actually, there is much significant evidence to suggest that predator dependence in the functional response occurs quite frequently in laboratory and natural systems [18]. One of the more

☆ This work was supported by research grants from the Catholic University of Daegu in 2013.
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0378-4754/$36.00
http://dx.doi.org/10.1016/j.matcom.2013.05.011

widely known is the Hassell–Varley functional response[8]. A general predator–prey system with Hassell–Varley type functional response may take the following form:

$$\begin{cases} x'(t) = ax(t)\left(1 - \dfrac{x(t)}{K}\right) - \dfrac{cx(t)y(t)}{x(t) + by^{\gamma}(t)}, \\ y'(t) = -dy(t) + \dfrac{ex(t)y(t)}{x(t) + by^{\gamma}(t)}, \gamma \in (0, 1), \\ (x(0^+), y(0^+)) = (x_0, y_0) = \mathbf{x}_0, \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ stand for prey and predator density, respectively. The constants a, K, c, b, e and d are positive that stand for prey intrinsic growth rate, carrying capacity, capturing rate, half saturation constant, maximal predator growth rate, predator death rate, respectively. A scenario that would lead to (1.1) would occur if each prey encountered by a predator group were shared among the predators in the group. In a typical predator–prey interaction where predators do not form groups, one can assume that $\gamma = 1$, producing the so-called ratio-dependent predator–prey dynamics. From [6], it is reasonable to assume that γ equals 1/3 for terrestrial predators and 1/2 for aquatic predators that respectively form a fixed number of tight groups. Specially, system(1.1) with $\gamma = 0$ is a predator–prey system with Holling type II functional response. Mathematically, systems (1.1) with $\gamma = 0$ or 1 can be viewed as limiting cases of system (1.1).

There are number of factors in the environment to be considered in predator–prey systems. One of the important factors is impulsive perturbation such as fire, flood, etc., that are not suitable to be considered continually. These impulsive perturbations bring sudden change to the system. For example, consider the interaction between crops and locusts in a local region. Once a year or once several years, a large amount of locusts may invade into the region and cause damage to the crops together with the local locusts. It is natural to assume that these perturbations act instantaneously, that is, in the form of impulse. With the ideas of impulsive perturbations, in this paper, we consider the following predator–prey system with periodic constant impulsive immigration of the predator.

$$\begin{cases} \left.\begin{aligned} x'(t) &= ax(t)\left(1 - \dfrac{x(t)}{K}\right) - \dfrac{cx(t)y(t)}{x(t) + by^{\gamma}(t)}, \\ y'(t) &= -dy(t) + \dfrac{ex(t)y(t)}{x(t) + by^{\gamma}(t)}, \end{aligned}\right\} \quad t \neq nT, \\ \left.\begin{aligned} x(t^+) &= x(t), \\ y(t^+) &= y(t) + p, \end{aligned}\right\} \quad t = nT, \\ (x(0^+), y(0^+)) = (x_0, y_0) = \mathbf{x}_0, \end{cases} \tag{1.2}$$

where $0 < \gamma < 1$, T is the period of the impulsive immigration or stock of the predator and p is the size of immigration or stock of the predator. Such system is an impulsive differential equation whose theory and applications were greatly developed by the efforts of Bainov and Lakshmikantham et al. [4,12]. Moreover, the theory of impulsive differential equations is being recognized to be not only richer than the corresponding theory of differential equations without impulses, but also represents a more natural framework for mathematical systeming of real world phenomena. In recent years, systems with sudden perturbations have been intensively researched [2,3,13–15,19–28].

The main purpose of this paper is to investigate the dynamics of system (1.2). In the next section, we introduce some notations which are used in this paper. We study qualitative properties of system (1.2) in Section 3. So, we show the local stability of prey-free periodic solutions and give a sufficient condition for the permanence of system (1.2) by applying the Floquet theory. By using bifurcation theory, we show that the existence of a nontrivial periodic solution under some conditions. In Section 4, we numerically investigate that the effects of impulsive perturbations on inherent oscillation. Finally, we have a conclusion in Section 5.

2. Preliminaries

Firstly, we give some notations, definitions and Lemmas which will be useful for our main results.