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Original article

Indirect Inference in fractional short-term interest rate diffusions

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Abstract

In this article we discuss the estimation of continuous time interest rate models driven by fractional Brownian motion (fBm) using discretely sampled data. In the presence of a fractional Brownian motion, usual estimation methods for continuous time models are not appropriate since in general fBm is neither a semimartingale nor a Markov process. In this context, we discuss the use of simulation-based Indirect Inference.

Keywords: Stochastic differential equations; Fractional Brownian motion; Indirect Inference

1. Introduction

The use of continuous time models in finance, started with the seminal work of [6], allows the use of tools of probability and stochastic process to asset pricing. The principle of no-arbitrage pricing, introduced by [51,52], can be resumed as an imposition of a set of restrictions in stochastic processes measured in continuous time which does not allow for the existence of risk free profits.

Delbaen and Schachermayer [28] show that no-arbitrage pricing is only possible in processes know as semimartingales, and processes that do not belong to this class cannot be used as innovation process in financial asset modeling. However, recent articles (discussed in Section 2) show that in the presence of transaction costs and restrictions on the set of admissible strategies, there are processes more general than semimartingales which can be used as price process, and are consistent with the no-arbitrage principle. In particular, we show that the fractional Brownian motion (fBm) is one of the processess which are consistent with no-arbitrage under appropriate restrictions. This process is a generalization of Brownian motion and allows for dependent increments and long memory. Since the increments of this process are dependent, it is not a Markov process. Also, except for the particular case where the process reduces to the standard Brownian case, this process is not a semimartingale.

We discuss the implications of fBm in the estimation of stochastic differential equations using discretely sampled data. In this situation, most of the estimators proposed for continuous time models using discrete data cannot be applied, since the violation of Markov property prevents the construction of likelihood functions in analytical or approximate forms. However, we can use the principle of indirect inference [44] to construct an estimator for stochastic differential equations based on fBm. The principle of indirect inference uses an auxiliary model based on an approximate

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and analytically tractable specification of the model. The correction of the inconsistency generated by the incorrect specifications is performed through Monte Carlo simulation. The principle of indirect inference can be used in this context of non Markovian/non semimartingale stochastic differential equations since it does not demand the exact likelihood function of the process, which is of infinite dimension in the fBm case. We show how to implement the indirect inference principle for stochastic differential equations driven by fBm, discussing with special attention the computational difficulties in the implementation of this estimator.

This article is organized as following: in Section 2, we present a short review of the connections between the principle of no-arbitrage and semimartingales and show a brief description of some properties of fBm. Section 2.1 describes stochastic differential equations driven by fBm discussed in the article, generalizing the univariate CIR process of [23] and the multivariate Wishart process of [18]. In Section 3, we review the related literature on estimation of stochastic differential equations, and discuss the limitations of the existing estimators in the presence of an fBm. In Section 4, we describe the proposed indirect inference estimator and the computational difficulties involved, Section 5 shows the properties of indirect inference estimator and the GMM auxiliary model by Monte Carlo simulations for CIR-fBM and Wishart–fBm processes. Section 6 shows the real data applications using a series of Canadian short-term interest rates. In Section 7, we have our final comments. Some further discussion, specification analysis and other empirical applications can be found in the Appendix.

2. Arbitrage, semimartingales and fractional Brownian motion

The principle of pricing by no-arbitrage [51,52] states that the price of an asset can be calculated as the price of a replicating portfolio that reproduces the discounted payoff from the stochastic process of interest.[1] Defining a probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t>0}, P)$ with $d+1$ assets, a particular asset $R=(R_t)_{t>0}$ being a bank account representing the risk free asset, $\mathcal{F}_{t-1}$ measurable, and a vector $S=(S^1, \ldots, S^d)$ of risky assets prices with dimension d, with $S^i=(S^i_t)_{t\geq 0}$, $\mathcal{F}_t$ -measurable, the main result of no-arbitrage pricing, known as fundamental theorem of asset pricing, claims that a market is free of arbitrage if and only if there exists (at least one) measure of probability Q equivalent to the measure P such that the discounted sequence $(S_t/R_t)_{t>0}$ is a Martingale process with respect to the measure Q.[2] The result in [28] is clear – no-arbitrage pricing is only possible for semimartingales processes. Thus, more general classes of processes that are not semimartingales cannot in principle be used as price processes in finance. This limitation can be very restrictive, since there are processes that are not semimartingales with interesting features (for example, some form of dependence in the increments of process) which could be used as price processes in finance.

However, there is strong evidence of the presence of long memory effects in financial assets. Backus and Zin [7] indicate that the volatility structure of bond yields has a hyperbolic pattern of decay compatible with the presence of long memory, and that departures from the short memory structure can substantially improve model performance. Meade and Maier [71] find evidence of long memory for interest rates series of the 3-month Euro Deposit rates for 7 of the 10 countries analyzed in the sample, and out-of-sample forecasting performance for the year ahead of the long memory models was significantly better than simple random walk models. In [31] the results obtained for a series of overnight and one-week Eurodollar rates indicates that the data support deviations from the Markovian assumption and the long memory parameter is small but precisely estimated. This evidence of long memory is also relevant for the modeling of the conditional volatility, as shown in [16,56], the realized volatility [68], and the bid-ask spreads [45] among many other applications in finance.

A process with interesting characteristics to represent prices is the fBm.[3] The fBm, introduced in [66] and formalized by [69] is the simplest stochastic process in continuous time with long memory. In this process the increments are

[1] See [90, Chapter 5] and [29] for further mathematical explanations.

[2] This definition is mathematically informal. In Asset Pricing Fundamental Theorem' rigorous definition [28], the existence condition for an Equivalent Martingale Measure is the validity of the condition knows as No Free Lunch with Vanishing Risk. No Arbitrage and No Free Lunch with Vanishing Risk are equivalent when we have a finite sample space Ω.

[3] A compilation of results about fBm can be found in [73].