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Original article

A detailed comparison of value at risk estimates

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Abstract

This work investigates the performance of different models of value at risk. We include several methods (parametric, historical simulation, Monte Carlo, and extreme value theory) and some models to compute the conditional variance. We analyze several international stock indexes and examine two types of periods: stable and volatile periods. To choose the best model, we employ a two-stage selection approach. The result indicates that the best model is a parametric model with conditional variance estimated by an asymmetric GARCH model under Student's *t*-distribution of returns. This paper shows that parametric models can obtain successful VaR measures if conditional variance is estimated properly.

Keywords: Value at risk; Parametric model; Extreme value theory model; GARCH model; Risk management

1. Introduction

One of the most important tasks for financial institutions is evaluation of market risk exposure. A possible way to measure this risk is to evaluate losses likely to incur when the price of the portfolio's assets declines. This is the purpose of the value at risk (VaR) methodology. Because the Basel Committee on banking supervision at the Bank for International Settlements requires financial institution to meet capital requirements on base VaR estimates, this methodology has become a basic market risk management tool by financial institutions.

Consequently, a growing body of literature has (1) proposed new models for VaR estimation, attempting to improve upon the existing ones, and (2) compared the performance of alternative proposed models. This comparison has two dimensions: (i) some authors compare alternative methods of VaR; and (ii) others compare a specific VaR method under alternative modeling of the conditional variance. We contribute to the second stream of this literature with a detailed statistical comparison of several VaR methods under various models of the conditional variance.

In the first dimension, recent papers compare the performance of some VaR methods, such as historical simulation, filtered historical simulation, Monte Carlo simulation, parametric methods and methods based on the extreme value theory. These papers do not provide conclusive results. Some papers (see Bao et al. [3], Consigli [7], Danielsson [8], Sarma et al. [36], Danielsson and Vries [9], among others) find evidence in favor of parametric methods, while others remark on the good results obtained by the filtered historical simulation (see Giannopoulos and Tunaru [15], Zikovic and Aktan [39], among others). More recently, studies (see Nozari et al. [32], Zikovic and Aktan [39], Gençay and

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Selçuk [14], among others) show that the models based on conditional extreme value theory performs the best for forecasting VaR.

The second dimension is related to the appropriate modeling of the conditional variance of returns. Some authors show that VaR accuracy depends on volatility models. In the parametric framework, some papers show that when more sophisticated volatility models are considered and/or assume higher moments of the density function of returns are not constant, the VaR estimates are more accurate (see Ergun and Jun [12], Mancini and Trojani [24], Polanski and Stoja [34], Ñíguez [33], Gonzalez-Riviera et al. [16], Bali et al. [2], among others).

Following a different approach, in the framework of the parametric method, McAleer et al. [25] propose a learning strategy or decision rule that consists of choosing from among different alternative VaR models, and McAleer et al. [26] suggest using a combination of VaR models to obtain a crisis-robust risk management strategy.

The aim of this paper is to compare different VaR methodologies. The main differences with the previous literature are as follows: (1) in this comparison we consider a more exhaustive set of methods – historical and Monte Carlo simulation, the parametric approach and conditional and unconditional extreme value theory methods; (2) when conditional variance needs to be modeled, we include several models (one of them being the asymmetric GARCH model under both a normal and a Student's t-distribution of returns, which allows for the leverage effect usually observed in financial returns); (3) we analyze the VaR performance in stable and volatile periods; and (4) we evaluate VaR methods on the basis of two criteria – the point of view of their forecasting accuracy and the point of view of a reasonable loss function that might represent investors' or regulators' concerns. Notwithstanding some criticism of the reliability of the parametric approach, we find evidence that this method works well to estimate value at risk when this method is used under an appropriate specification for conditional volatility and appropriate distribution for the returns.

In the next section, we present the VaR methods we use in the paper. In Section Three, the data used for estimation and forecasting VaR are presented. In the fourth section, we review the statistical test and the loss function that used to evaluate the performance of the VaR estimates. The fifth section presents the empirical results. The last section includes the main conclusions.

2. Characteristics of VaR models

Let $r_1, r_2, r_3, \ldots, r_n$ be identically distributed independent random variables representing financial returns. Use $F(r)$ to denote the cumulative distribution function $F(r) = Pr(r_t < r | \Omega_{t-1})$ conditional on the information set Ω_{t-1} available at time $t-1$. Assume that $\{r_t\}$ follows the stochastic process

$$r_t = \mu + \sigma_t \varepsilon_t \quad \varepsilon_t \sim iii(0, 1) \tag{1}$$

where $\sigma_t^2 = E(\varepsilon_t^2 | \Omega_{t-1})$ and ε_t has conditional distribution function $G(\varepsilon)$, $G(\varepsilon) = Pr(\varepsilon_t < \varepsilon | \Omega_{t-1})$. It can be assumed that $\sigma_t = \sigma$ for all t or that σ_t has a probability density $Pr(\sigma_t | \Omega_{t-1})$. In this paper, we consider the latter. The VaR with a given probability $\alpha \in (0, 1)$, denoted by VaR(α), is defined as the α quantile of the probability distribution of financial returns:

$$F(\text{VaR}(\alpha)) = Pr(r_t < \text{VaR}(\alpha)) = \alpha \quad \text{or} \quad \text{VaR}\,(\alpha) = \inf\{v \,|\, P(r_t \leq v) = \alpha\} \tag{2}$$

There are two methods to estimate this quantile: (1) inverting the distribution function of financial returns $F(r)$ and (2) inverting the distribution function of innovations $G(\varepsilon)$. With regard to the latter, it is also necessary to estimate σ_t^2.

$$\text{VaR}\,(\alpha) = F^{-1}(\alpha) = \mu + \sigma_t G^{-1}(\alpha) \tag{3}$$

Therefore, a VaR model involves the specification of $F(r)$ or σ_t^2 and $G(\varepsilon)$. The historical simulation, Monte Carlo simulation and unconditional extreme value theory approaches focus on estimating $F(r)$, while the parametric and conditional extreme value theory approaches focus on estimating $G(\varepsilon)$. These functions can be estimated parametrically (the parametric method), nonparametrically (historical simulation) or by focusing on tail behavior (extreme value theory). We describe the general characteristics of the methods that we compare in this paper.[1]

[1] In this comparison, we do not include other methods, such us filtered historical simulation, the CaViar method or extensions of the latter (such as Yu et al. [38]). Although including these other methods would be interesting, we have left the task for future research.