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Original Article

Typical bifurcation scenario in a three region identical New Economic Geography model

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Abstract

We study global dynamics of the New Economic Geography model which describes spatial distribution of industrial activity in the long run across three identical regions depending on the balancing of agglomeration and dispersion forces. It is defined by a two-dimensional piecewise smooth map depending on four parameters. Based on the numerical evidence we discuss typical bifurcation scenarios observed in the model: starting from the symmetric fixed point (related to equal distribution of the industrial activity in all the three regions) two different scenarios are realized depending on whether the transportation cost parameter is increased or decreased. Emergence of the Wada basins of coexisting attractors leading to the so-called final state sensitivity is discussed, as well as final bifurcation of the chaotic attractor.

Keywords: New economic geography model; Two-dimensional piecewise smooth map; Bifurcation scenarios; Milnor attractor; Wada basin

1. Introduction

We investigate global dynamics of a discrete time *New Economic Geography* (NEG) model introduced in [4], which describes spatial distribution of industrial activity in the long run across *three identical regions*. The NEG approach aims to explain how industrial economic activity tends to spread or agglomerate across space. Originating from Krugman's Core-Periphery model [16], several NEG model variants have been proposed in the literature. The main ingredients are increasing returns and goods differentiation in the manufacturing production; transport costs which determines geographical distance/separation between two regions; and, finally, factor mobility between regions driven by an economic incentive and described as a continuous time dynamic process. In the C-P model, the migration decisions of workers depend on real wage differentials. Workers enter in the production process both as a fixed and as a variable factor. In the footloose entrepreneurs (FE) model, proposed by Forslid and Ottaviano [7], the mobility hypothesis involves entrepreneurs/human capital. Entrepreneurial activity enters in production only as a fixed component, simplifying notably the analytical structure. NEG analyses highlight that in the long run several regional distributions of economic activity are possible. It depends on which initial conditions and/or parameters will actually prevail. In more technical

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terms, coexistence of long run attractors and the nature and structure of the basins of attraction of long-term positions are crucial results in these analyses.

We modify the FE model in two important respects: first, we increase the number of regions from two to three; and second we frame the migration process in discrete time. Following Fujta and Thisse [11] by increasing the number of regions it is possible to capture spatial spillover effects that do not emerge in a 2-region context.[1] The discrete time specification takes into account that factor mobility is associated with considerable delays that continuous time excludes a priori. As we shall see, the possible spatial distribution patterns of economic activity, as well as the degree of complexity of the basins of attraction, are substantially increased.

The considered NEG model is defined by a two-dimensional (2D for short) piecewise smooth continuous noninvertible map Z which depends on four parameters. Analysis of the local stability of the fixed points of Z as well as some preliminary results on its global dynamics are presented in [4]. Purpose of the present paper is to clarify which bifurcation scenarios can be realized in the system depending on parameters. Besides nonsmoothness and noninvertibility, the peculiarity of the map Z is related to its *symmetry* in the phase plane with respect to the main diagonal. Due to this property any invariant set of the map Z is either symmetric itself or there exists other invariant set symmetric to the considered one. Coexistence of different attractors leads to the problem of definition of their basins of attraction. It is well known that a basin of attraction can be a simply- or multiply-connected set; it can consist of finite or infinite (countable or uncountable) number of disjoint sets (see, e.g., [18,19]). It is also known that the basin boundaries, which are invariant sets separating one basin from others, can be a smooth curve (e.g., stable manifold of a saddle cycle), or it can have a fractal structure. In fact, a basin boundary is fractal if it contains a transversal homoclinic point. We show that in the map Z one of the most complex kind of basins, so-called *Wada basins*, can be observed [14,21–23]. The importance of such kind of basins is related to the fact that fractal and Wada basins lead to a *final state sensitivity* [13], namely, for a specific parameter setting and initial condition, no reliable computation can be made to predict the system's asymptotic behaviour.

The paper is organized as follows. In Section 2 we recall the basic economic framework of the considered NEG model. In Section 3.1 we discuss dynamics of 1D maps embedded into the 2D map Z, namely, we show that in the phase plane of the map Z there are several invariant straight lines on which the 2D map Z is reduced to 1D piecewise smooth maps which can have quite complicated dynamics, including coexistence of several attractors and chaos. Moreover, for certain parameter values an additional flat branch can appear in the definition of these maps due to which locally repelling fixed points become Milnor attractors. In Section 3.2 we illustrate, by means of 1D and 2D bifurcation diagrams, possible bifurcation scenarios realized in the system when the symmetric fixed point of the map Z (related to equal distribution of the industrial activity in all the three regions) loses its stability. In particular, we show that codimention-2 transcritical and flip bifurcations of the fixed point occur depending on whether the transportation cost parameter is increased or decreased. The bifurcation scenario which follows after the first flip bifurcation includes coexisting attractors which can have Wada basins, examples of which are presented in Section 3.3. The final bifurcation of a chaotic attractor occurring due to its contact with the basin boundary is discussed in Section 3.4. Several final remarks are given in Section 4.

2. The economic framework

2.1. Main assumptions

The economy is composed of three trading regions ($r = 1; 2; 3$), two production sectors, agriculture (A) and manufacturing (M) and two factors of production, unskilled labour (L) and entrepreneurs (N). The three regions are identical ("symmetric" as this is called in the context of the New Economic Geography) with respect to the following: they are characterized by the same technology, tastes and transport costs; and they are equally endowed with unskilled workers, who are inter-sectorally mobile but inter-regionally immobile. Entrepreneurs, instead, are allowed to migrate from

[1] Notwithstanding the early contribution by Krugman [15] (see also [10]), until recent times there has been a scant attention devoted to multi-regional NEG models, an exception is [6]. Concerning 3-region NEG models, recent contributions include [2] and [12], presenting continuous time stability analyses of the C-P and FE models; and [4] that put forward a discrete time FE model, exploring the fixed points local stability properties and developing a preliminary global stability analysis. Concerning 2-region NEG models framed in discrete time, these have been proposed in [5] and in [3], where the C-P model and the FE model are respectively reformulated.