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Original Article

Dynamic monopoly with multiple continuously distributed time delays

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Abstract

Two time delays are assumed in a boundedly rational monopoly. The characteristic equation is derived for the general case, and a complete stability analysis is conducted both analytically and numerically in two special cases. In the first case, wherein the continuously distributed time delays have different weights, it is shown that a monopoly equilibrium is destabilized to generate a limit cycle via Hopf bifurcation. In the second case in which one delay is continuously distributed and the other is fixed, it is demonstrated that the stability of the monopoly equilibrium can change finite number of times and eventually becomes periodically or aperiodically unstable. It is of interest to notice that the two delay dynamics can be qualitatively different from the one delay dynamics.

JEL classification: C62; C63; D21; D42

Keywords: Boundedly rational; Continuously distributed time delay; Fixed time delay; Hopf bifurcation; Complex dynamics

1. Introduction

This study analyzes a dynamic model of a boundedly rational monopoly in continuous-time scale and investigates the destabilizing effects caused by time delay in the demand function.[1] If the delay is the result of the process of information collection and its length is known, then models should include *fixed time delay*. If the length of the delay is uncertain, or the monopoly wants to react to an average of past information instead of reacting to sudden market changes, then *continuously distributed time delays* have to be used [1,2,4]. These two types of models can be combined if the firm believes in a fixed delay but still wants to react to average past demand data.

This paper is a continuation of [8] in which the dynamic monopoly model is constructed under a single continuously distributed time delay (continuous delay henceforth). The monopoly stationary point is shown to bifurcate to a limit cycle through Hopf bifurcation when it loses stability. It is also a complement of [7] in which the continuous delay is replaced with a fixed time delay (fixed delay henceforth). It is demonstrated that Hopf cycles emerge under one fixed

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[1] Simon's bounded rationality is assumed here. It is not related to the cognitive process of the monopolist. This is pointed out by a reviewer.

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delay and so does the complex dynamics involving chaos under two fixed delays. The point is that periodic behavior emerges if the quantity adjustment process has one time delay, regardless of the delay being continuous or fixed, while aperiodic behavior can be achieved if it contains two fixed delays. The natural inference from these results is that erratic behavior might be expected if multiple continuous delays are involved. The main purpose of this study is to confirm this fact in the boundedly rational monopoly model. We mention however that this is not the case in general (see, for example [6] or [3]).

The principal impetus is provided by the dynamic analysis of the boundedly rational monopoly with discrete-time scale in [10] and more recently in [9]. A gradient rule is assumed, in both studies, when production is increased if a change in profit is positive, decreased if negative and constant if zero. A cubic demand function with an inflection point is assumed in the former and this particular nonlinearity is shown to be a main source for chaotic attractor. On the other hand, a cubic demand without an inflection point is assumed in the latter and nonetheless stability is violated to chaos through the familiar period-doubling cascade. In this study, the same gradient dynamics is considered under different conditions, namely, the demand function is linear, a continuous-time scale is adopted and the growth rate of output is proportional to the marginal change in expected profit. Special attention is given to the destabilizing effect caused by two continuous delays.

The paper is organized as follows. Section 2 presents a basic monopoly model with continuous delays. Section 3 examines the case in which two continuous delays have different weighting functions. Section 4 investigates the special case in which continuous and fixed delays coexist. Finally concluding remarks are given in Section 5.

2. Delay monopoly

2.1. Basic model

Consider an output decision problem of a boundedly rational monopolistic firm which produces output q with marginal cost c. The price function is assumed to be linear

$$f(q) = a - bq, a, b > 0.$$

It is further assumed that the firm does not want to react to sudden market changes, so instead of the most current price information, an average of past prices is used in the adjustment process. Because of the linearity of the price function it is equivalent to the use of an average of past output data q^e in the adjustment scheme. Then the corresponding marginal profit, $\partial\pi/\partial q$, is given as $a - c - 2bq^e$ which generates the approximating gradient dynamics

$$\frac{\dot{q}(t)}{q(t)} = \alpha(a - c - 2bq^e(t)) \tag{1}$$

with $\alpha > 0$ being an adjustment coefficient, furthermore t denotes a point of continuous time and the dot over a variable means a time derivative. Eq. (1) implies that the growth rate of output is adjusted in proportion to the average marginal profit. In constructing myopic optimal dynamics, global information is required about the profit function, however, in applying gradient dynamics, only local information is needed. The dynamic equation is written as

$$\dot{q}(t) = \alpha q(t)\left[a - c - 2bq^e(t)\right]. \tag{2}$$

Since $q(t) = q^e(t)$ holds at a stationary point, Eq. (2) has two stationary points; the zero trivial point $q(t) = 0$ and a nontrivial point

$$q^M = \frac{a - c}{2b}$$

where $a > c$ is assumed to ensure that the nontrivial point is positive. We call q^M a *monopoly equilibrium*. Dynamic behavior of Eq. (2) depends on the formation of averaging past data. With continuous-time scale, time delays can be modeled with a continuous or fixed delay. As is mentioned in the Introduction, dynamic analysis has been done under the assumption of a single continuous delay, one and two fixed delays. In this study we adopt multiple continuous delays and draw attention to the destabilizing effects of continuous delays having different weights. In addition we will investigate the limiting case when one delay is continuous and the other is fixed.