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Takeoff vs. stagnation in endogenous recombinant growth models

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Abstract

This paper concludes the study of transition paths in the continuous-time recombinant endogenous growth model by providing numerical methods to estimate the threshold initial value of capital (a Skiba-type point) above which the economy takes off toward sustained growth in the long run, while it is doomed to stagnation otherwise. The model is based on the setting first introduced by Tsur and Zemel and then further specified by Privileggi, in which knowledge evolves according to the Weitzman recombinant process. We pursue a direct approach based on the comparison of welfare estimations along optimal consumption trajectories either diverging to sustained growth or converging to a steady state. To this purpose, we develop and test three algorithms capable of numerically simulating the initial Skiba-value of capital, each corresponding to initial stock of knowledge values belonging to three different ranges, thus covering all possible scenarios.

Keywords: Knowledge production; Endogenous recombinant growth; Transition dynamics; Skiba point; Hamilton–Jacobi–Bellman equation

1. Introduction

This paper provides a further contribution to the study of the two-sector continuous-time endogenous growth model introduced by Tsur and Zemel [24] in which knowledge evolves according to the Weitzman [25] recombinant process. Given any feasible initial stock of knowledge, we provide three numeric algorithms capable of approximating the corresponding critical initial capital value above which the economy "takes off" toward an asymptotic balanced growth path (ABGP), while it is led toward a steady point–*i.e.*, to stagnation in the long run–whenever the initial capital lies below such threshold. To this purpose we elaborate on the functional forms introduced by Privileggi [19], which are suitable to 'detrend' the model and thus obtain a closed form for the ODE defining the optimal policy that, in turn, can be approximated with a sufficient degree of accuracy by means of a projection method discussed in [20].

Weitzman [25] departs from the mainstream endogenous growth literature[1] flourished after the original works of either Romer [21,22]–based on technology spillovers/externalities–or Grossman and Helpman [8] and Aghion and Howitt [2]–building on the Schumpeterian tradition of the creative-destruction process involved in innovation activities–by focusing on two peculiar elements that drive knowledge generation: the process of creation of new

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[1] For recent comprehensive surveys see [4,1] and, more oriented toward the creative-destruction Schumpeterian-style models, [3].

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ideas and the resources needed to turn these ideas into "productive" knowledge. The evolution of ideas is assumed to follow a *recombinant mechanism*: existing ideas are combined (matched) to generate new ideas. The number of possible matchings is a combinatorial function of the number of existing ideas that spontaneously would give rise to an unrealistic over-exponential growth. As a matter of fact, such explosive dynamic is contained by the fact that turning potentially fruitful ideas into useful knowledge requires physical resources, whose optimal allocation by a social planner has been first analyzed by Tsur and Zemel [24] in a continuous-time setting.

We consider a specification of the model in [24] where the probability of a successful matching among existing seed ideas has a hyperbolic form, a composite final good is produced in a competitive sector by means of a Cobb–Douglas function using the stock of knowledge and physical capital as input factors, and the representative household has a CIES utility function. A social planner efficiently maximizes the discounted utility of a representative consumer over an infinite time horizon by directly financing new knowledge production through a tax levied on the households; at each instant new knowledge is produced by an independent R&D sector under the supervision of the social planner. Hence, we are pursuing a first-best, social planner-type equilibrium approach, setting aside all issues regarding incentives to innovate, spillovers, externalities, etc., and the related scale effects involved by knowledge production.[2]

This hyperbolic-Cobb–Douglas-CIES specification of the model allows for a closed-form ODE defining the optimal transition dynamics (see [19]) along a characteristic curve in the knowledge-capital state space that will be labeled as (transitory) *turnpike* when the conditions for sustained long-run growth provided by [24] are met. The solution of such ODE is numerically approximated through an appropriate projection method (see [20]) and can be used to compute, by means of a finite-difference, Runge–Kutta method, the optimal time-path trajectories of the stock of knowledge, capital, output and consumption, as well as their transition growth rates, along the turnpike. However, whenever the initial capital is different than its unique value on the turnpike, different types of transition paths appear; they can either reach a point on the turnpike in a finite time period and then continue along the turnpike itself toward sustained growth, or can converge to a steady state which is a point on another characteristic curve in the knowledge-capital state space that will be called the *stagnation line*.

The aim of this paper is to thoroughly investigate the latter type of (initial) transitions. Tsur and Zemel [24] showed that, for each given initial stock of knowledge, there corresponds a unique critical value for the initial capital such that for any value above this threshold the economy will first follow a path toward the turnpike and then, along a path evolving along the turnpike itself, toward sustained growth along a ABGP. Conversely, when the initial capital is below such threshold, the process generating new knowledge does not take off and the economy eventually dies in stagnation by converging asymptotically to steady values for both knowledge and capital on the stagnation line. The properties of this threshold value are akin to those first discussed by Skiba [23]; hence we shall refer to this point as the *Skiba-point*.

We first develop a numerical method (Algorithm 1) that computes the Skiba-point *on the turnpike*, labeled as (A_m, k_m^{sk}), by equating the welfare when sustained growth is triggered with the welfare associated to a path leading toward stagnation, starting from the same point (A_m, k_m^{sk}). Next, we consider initial values of the stock of knowledge, A_0, which lie on the left of A_m and build a more complex Bisection method (Algorithm 2), again with the goal of matching the welfare when taking off toward the ABGP with the welfare of the economy converging to a steady state, to find the Skiba-point when the economy starts on an initial capital, k_0, lying *above the turnpike* value corresponding to A_0. Finally, we focus on initial values A_0 lying to the right of A_m; in this case, to estimate the Skiba-point we propose another Bisection method (Algorithm 3) with the aim of equating the welfare generated by the trajectory that starts from an initial capital, k_0, *below the turnpike*, climbs up toward the turnpike, reaches it in a finite time period, and then keeps following it thereafter toward the ABGP, with that produced by the trajectory converging to a steady state starting from the same initial point (A_0, k_0).

All optimal trajectories are estimated through a mix of *projection methods* and *Runge–Kutta* type algorithms. First a projection method–based either on *OLS* or on *Orthogonal Collocation* and with a *residual function* defined by means of *Chebyshev polynomials* (see, *e.g.*, Chapter 11 in [16], Chapter 6 in [12], or Paragraph 5.5.2 in [18])–is applied to the ODE defining the optimal policy. The approximation thus obtained can then be used in a Runge–Kutta method to generate all transition time-path trajectories. Welfare estimates along the turnpike or toward stagnation are performed through direct computation of the value function by means of the *Hamilton–Jacobi–Bellman equation* in which the derivative of the value function is calculated through the celebrated Benveniste and Scheinkman [5] result as the

[2] For a general and exhaustive discussion on all these issues see [13,14] and [15].