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Original article

Complex attractors and basins in a growth model with nonconcave production function and logistic population growth rate

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Abstract

In this paper we study a discrete-time growth model of the Solow type with nonconcave production function where shareholders save more than workers and the population growth dynamics is described by the logistic equation. We prove that the resulting system has a compact global attractor and we describe its structure. We also perform a mainly numerical analysis to show that complex features are exhibited, related both to the structure of the coexisting attractors and to their basins. The study presented aims at showing the existence of complex dynamics when the elasticity of substitution between production factors is not too high (so that capital income declines) or the parameter in the logistic equation increases (so that the amplitude of movements in the population growth rate increases).

Keywords: Triangular system; Critical curves and absorbing areas; Local and global dynamics; Economic growth and population dynamics

1. Introduction

The standard one-sector Solow–Swan model (see [34,35]) represents one of the most used framework to describe endogenous economic growth; it shows that the system monotonically converges to the steady state, so neither cycles nor complex dynamics can be observed. In order to investigate the possibility of complex dynamics to be exhibited in optimal growth models, many authors have studied the question whether the different saving propensities of two groups (labor and capital) might influence the final dynamics of the system.

The question of differential savings between groups of agents was originally posed within the Harrod–Domar model of fixed portion [23]. Obviously different but constant saving propensities make the aggregate saving propensity non-constant and dependent on income distribution so that multiple and unstable equilibria may occur. However, qualitative dynamics is still simple.

Bohm and Kaas [7] investigated the discrete-time neoclassical growth model with constant but different saving propensities between capital and labor using a generic production function satisfying the *weak Inada conditions*, i.e. $\lim_{k_t\to\infty} f(k_t)/k_t = 0$, $\lim_{k_t\to 0} f(k_t)/k_t = \infty$. The authors showed that instability and topological chaos can be generated in this kind of model.

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Following the contribution by Bohm and Kaas a number of studies have considered the following two questions. First of all, the elasticity of substitution between production factors plays a crucial role in the theory of economic growth since it represents one of the determinants of the economic growth level. Hence the question of how the use of different production functions may affect the qualitative and quantitative dynamics in the long term becomes crucial. Secondly, the assumption that labor force grows at a constant rate is quite unrealistic since it implies that population grows exponentially. Hence different iteration schemes that are able to describe the evolution of the population growth rate must be considered.

Several papers have been proposed to give an answer to the first question. Brianzoni et al. [8–10] investigated the neoclassical growth model in discrete time with differential savings and endogenous labor force growth rate while assuming Constant Elasticity of Substitution (CES) production function. The authors proved that multiple equilibria are likely to emerge and that complex dynamics can be exhibited if the elasticity of substitution between production factors is sufficiently low. Later, Tramontana et al. [36] studied the bifurcations related to the presence of a discontinuity point in the map with Leontief technology, representing the limit case as the elasticity of substitution tends to 0. As a further step in this field, Brianzoni et al. [12] firstly introduced the Variable Elasticity of Substitution (VES) production function in the form given by Revankar [31]. The authors proved that the model can exhibit unbounded endogenous growth (differently from CES) and that the production function elasticity of substitution is responsible for the creation and propagation of complicated dynamics, as in models with explicitly dynamic optimizing behavior by the private agents. More recently, Brianzoni et al. [11] considered the nonconcave production function, as firstly formulated by [19,33], proving that similarly to what happens with the CES and VES production function, if shareholders save more than workers and the elasticity of substitution between production factors is low, then the model can exhibit complexity.

In order to give an answer to the second question, different iteration schemes have been introduced in economic growth models to describe the evolution of the population growth rate. Refs. [8,16] investigated the Solow–Swan growth model with labor force dynamics described by the Beverton–Holt (BH) equation (see [3]) while assuming CES production function; the results reached have been then generalized in [9] where a generic map having a unique positive globally stable fixed point was proposed to describe the evolution of the population growth rate. More recently the BH equation was also proposed in a growth model with VES production function (see [17]). While in the abovementioned works the authors considered simple population dynamic laws such that the population growth rate converges to a unique globally stable fixed point, in a number of papers the population growth evolution has been formalized with the well-known logistic map, able to exhibit more complex dynamics as cycles of every order or chaos. For instance in [10,15] the economic setup with CES production function was considered, while in [18] the VES production function was taken into account.

In the present paper we study the discrete time one sector Solow–Swan growth model with differential savings as in [7], while assuming that: (1) the technology is described by a nonconcave production function in the form proposed by Capasso et al. [14] and (2) the population growth dynamics is described by the logistic equation. The reasons for introducing these two assumptions are the following:

(1) For many economic growth models the production function is assumed to be non-negative, increasing and concave, and also to fulfill the so called *Inada Conditions*, i.e. $f(0)=0$, $\lim_{k_t\to 0} f'(k_t)=+\infty$ and $\lim_{k_t\to +\infty} f'(k_t)=0$. Let us focus on the meaning of condition $\lim_{k_t\to 0} f'(k_t)=+\infty$ from an economic point of view. Consider a region with almost no physical capital, that is there are no machines to produce goods, no infrastructure, etc. Then the previous condition states that it is possible to gain infinitely high returns by investing only a less amount of money. This obviously cannot be realistic since before getting returns it is necessary to create prerequisites by investing a certain amount of money. After establishing a basic structure for production, one might still get only less returns until reaching a threshold where the returns increase greatly to the point where the law of diminishing returns takes effect. In the economic literature this fact is known as *poverty trap*. In other words, one should expect that there is a critical level of physical capital having the property that, if the initial value of physical capital is lesser than such a level, then the dynamic of physical capital will descend to the zero level, thus eliminating any possibility of economic growth. Following this argument concavity assumptions provide a good approximation of a high level of economic development but it is not always applicable to less-developed countries. Thus it makes sense to assume that only an amount of money greater than some threshold will lead to returns. The first model with nonconcave production function was introduced by [19,33]. Following such works several contributions have then focused on the existence