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# Giant vortex structures in mesoscopic spherical type-II superconducting samples

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## Abstract

Giant vortex states which can occur in type-II mesoscopic spherical superconducting samples embedded in different materials have been studied within the framework of the nonlinear Ginzburg–Landau theory. The proposed method, which is based on presentation of the superconducting order parameter in a form of the spherical functions expansion gives the possibility to take into account different types of boundary condition.

The upper critical field of a superconducting spherical inclusion was calculated for different types of the three-dimensional boundary condition.

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## 1. Introduction

Vast development of nanotechnology changes our knowledge about physics of the vortex state [13,20,18,21,3,17,7,5,2,1,10,9,22,19,16]. Progress in nanotechnology stimulates studying the properties of mesoscopic superconductors, i.e. superconductors whose sizes are commensurable to the coherence lengths or the penetration depths. In this case magnetic properties of the superconducting samples will be quite different from such properties of the bulk samples, and the boundary of a sample is the key parameter; moreover, the boundary imposes new types of different vortex structures which can be realized in a superconducting sample. It is a well known fact that the triangular Abrikosov vortex lattice has the lowest energy in the bulk superconductor. So, to determine the type of a structure which will be realized in the superconducting sample it is necessary to determine which structure will have the lowest energy, i.e. the first step in calculation of such structures is the free energy analysis.

Classical vortices or well known Abrikosov vortices carrying a single vortex quantum and having topological singularities in the order parameter are not favorable energetically in mesoscopic superconductors. Moreover, just a few vortices can penetrate inside a mesoscopic superconductor whereas in a bulk superconductor the number of vortices can be numerous. So, geometrical sizes and shape of a sample are very important parameters and they determine the types of vortex structures which can be realized in the superconductors. There are many experimental and theoretical

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papers devoted to the subject. Now it is well known that there are two types of vortex structures in mesoscopic samples: multivortex structures with a spatial arrangement of singly quantized vortex states, and axially symmetric giant vortex states [23,16,6]. First structures are the analog of Abrikosov vortices in bulk superconductors, the second structures are multiquantized vortices with a single core in the centre. The above mentioned states have been investigated both theoretically and experimentally for disks, films and wires [23,16,6,15,12]. Phenomena of the superconductivity in spherical superconducting samples is less studied than in disks, films and wires. This problem is more complicated due to its explicitly expressed 3D nature and it is necessary to take into account bending of vortices due to the effect of the boundary. To study magnetic properties of vortex states in spherical superconductors three different approaches were used [11,4,28]. In the first one the authors have proposed the new interesting method based on the analogy between the 3D lattice of inclusions and 3D lattice of mesoscopic superconductors. This method allows one to take into account that the vortex lines are naturally curved due to strong surface effects and that the giant vortex structure can break up into some smaller giant vortices or singly quantized vortices. In the second approach the cylindrical symmetric vortex states in mesoscopic spherical superconductors were studied using a minimization procedure of the Ginzburg–Landau free energy [4]. The investigation was based on solving the linearized first Ginzburg–Landau equation by treating it as an eigenvalue problem. In this way the giant vortex states in superconducting spheres of different sizes have been calculated and the possible multivortex states built in a form of the linear combinations of two different giant vortices have been determined. Moreover, this method allows to calculate the stability region of the giant vortex states which can be realized in a spherical superconductor. The submicron superconducting spheres were also studied using a fully three-dimensional numerical treatment [28]. In this approach the distribution of the magnetic field inside and around the spherical sample has been calculated that allows to calculate magnetization of the sample and to distinguish the multi-vortex and giant-vortex states. The fulfilled studying covers all aspects of a superconducting spherical sample investigation except the case when a sample is embedded in metallic materials. In this case the situation is more complicated and requires separate consideration.

The aim of this paper is to study magnetic properties of the mesoscopic type-II superconducting spherical samples embedded in different materials. In this case it is necessary to use the most general type of the boundary condition—de Gennes boundary condition [27], which depends on the material property. The proposed calculation approach is based on the presentation of the superconducting order parameter in a form of the spherical functions expansion. This method allows to substantiate in the framework of the phenomenological Ginzburg–Landau theory that the giant vortices can occur in spherical mesoscopic superconductors, and investigate the giant vortex evolution at different types of the three-dimensional boundary condition.

## 2. Giant vortices in a mesoscopic spherical superconductor

Let us consider a spherical superconducting inclusion embedded in a material in the framework of the nonlinear Ginzburg–Landau approach. We consider a type-II superconductor with $\kappa \gg 1$, so the distortion of the magnetic field from the external field caused by the currents circulation around the inclusion is small. So, the magnetic field is considered as constant throughout the sphere, and the vector potential is chosen in the form of $\vec{A} = \frac{1}{2}[\vec{H} \otimes \vec{\rho}]$, $H$ is the external magnetic field, $\kappa$ is the Ginzburg–Landau parameter. Free energy in spherical coordinates with the origin in the centre of the sphere can be written as:

$$F = \int \mathrm{d}V\psi^{*}\Big(-\psi - \frac{2}{\rho}\frac{\partial\psi}{\partial\rho} - \frac{\partial^2\psi}{\partial\rho^2} - \frac{1}{\rho^2}\left(\frac{\cos\theta}{\sin\theta}\frac{\partial\psi}{\partial\theta} + \frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial\phi^2}\right) + \imath H\frac{1}{\rho\sin\theta}\frac{\partial\psi}{\partial\phi}$$
$$+\frac{1}{4}H^2\rho^2\sin\theta^2\psi + \frac{1}{2}\psi|\psi|^2\Big), \qquad (1)$$

where $\psi$ is the superconducting order parameter, $\rho = r/\xi$ is the dimensionless radius with the origin in the centre of the sphere, the angle $\theta$ is counted from the magnetic field direction, the magnetic field is chosen in the units of the second critical magnetic field $H_{c2}$, $\xi$ is the coherence length.