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Original article

Plane-wave solutions of a dissipative generalization of the vector nonlinear Schrödinger equation

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Abstract

The modulational instability of perturbed plane-wave solutions of the vector nonlinear Schrödinger (VNLS) equation is examined in the presence of multiple forms of dissipation. We establish that all constant-magnitude solutions of the dissipative VNLS equation are less unstable than their counterparts in the conservative VNLS equation. We also present three families of decreasing-in-magnitude plane-wave solutions to this dissipative VNLS equation. We establish that if certain forms of dissipation are present, then all exponentially-decaying plane-wave solutions with spatial dependence are linearly unstable while those without spatial dependence are linearly stable.
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Keywords: Nonlinear; NLS; Stability; Plane-waves; Dissipation

1. Introduction

Benjamin and Feir [1] showed that a uniform train of plane waves of moderate amplitude in deep water without dissipation is unstable with respect to a small perturbation of other waves traveling in the same direction with nearly the same frequency. Since that classic paper, a significant amount of research has been dedicated to studying the stability of plane waves on deep water. In 1968, Zakharov [28] derived the (scalar) nonlinear Schrödinger (NLS) equation

$$i\psi_t - \psi_{xx} + \psi_{yy} - |\psi|^2\psi = 0, \tag{1}$$

where $\psi = \psi(x, y, t)$ is a complex-valued function, as an approximate model of the evolution of plane waves of moderate amplitude in deep water without dissipation. Plane-wave solutions of the NLS equation are unstable [28,9,21]. Dysthe [12], Trulsen and Dysthe [25], and Trulsen et al. [26] derived higher-order asymptotic generalizations of the NLS equation and found that the plane-wave solutions of these equations are also unstable. Dhar and Das [10] showed that plane-wave solutions of a coupled system of higher-order generalizations of the NLS equation are also unstable. Onorato et al. [20] and Shukla et al. [24] showed that the plane-wave solutions of the vector nonlinear Schrödinger (VNLS) equation are unstable.

Lake and Yuen [17] and Lake et al. [18] conducted thorough examinations of physical experiments of one-dimensional surface patterns in which instability growth rates were measured and compared with rates predicted

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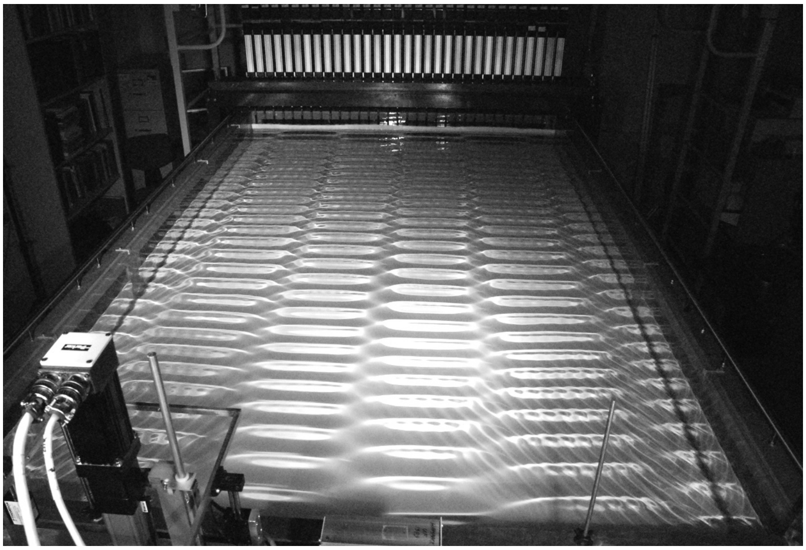

Fig. 1. A photograph of an apparently stable two-dimensional wave pattern [15].

by theoretical models. More recently, physical experiments of two-dimensional surface patterns were conducted by Kimmoun et al. [16], Hammack et al. [13], and Henderson et al. [14]. Surprisingly, many of the wave patterns in these two-dimensional experiments appeared to be stable. A photograph of an apparently-stable two-dimensional surface wave pattern is included in Fig. 1.

All of the theories mentioned above predict that plane-wave solutions are unstable while some of the experiments suggest that plane waves are stable. This apparent discrepancy can be explained by the fact that all of the aforementioned models are conservative and none include effects due to dissipation.

There are many generalizations of the NLS equation that contain dissipative terms including those in [19,2,22,27]. Segur et al. [23] studied a dissipative generalization of the NLS equation and showed that all plane-wave solutions are stable if a certain form of dissipation is present. Further, they showed that the dissipative theory agreed well with measurements from a series of physical experiments. Craig et al. [7] and Henderson et al. [15] generalized the Segur et al. [23] results to the vector dissipative NLS equation (i.e. a pair of nonlinearly coupled dissipative NLS equations). The photograph included in Fig. 1 is from an experiment modeled by a dissipative generalization of the vector NLS equation [15]. In this experiment, the undisturbed water depth is approximately 20 cm, the wave amplitude is approximately 0.4 cm, and the carrier wave frequency is 4 Hz. These parameters establish that the VNLS equation is an appropriate model equation.

In Section 1.1, we introduce a generalization of the vector NLS equation that contains multiple forms of dissipation as a model of two-dimensional surface wave patterns on deep water. This model is more general than the model contained in [7,15]. In Section 1.2, we present four families of plane-wave solutions to this equation, three of which decrease in magnitude.

In Section 2, we present the linear stability analysis for three of these families of solutions. We establish that all constant-magnitude solutions are “less unstable” than their counterparts in the conservative system. This means that all plane-wave solutions that are stable in the conservative system are also stable in the dissipative system. It also means that solutions that are unstable in the conservative system have smaller (or possibly zero) growth rates if certain forms of dissipation are present. Finally, we establish that if certain forms of dissipation are present, then all spatially-independent exponentially-decaying plane-wave solutions are stable and all spatially-dependent exponentially-decaying plane-wave solutions are unstable.

1.1. The vector dissipative NLS equation

In order to model the evolution of a uniform wave train of surface waves with a two-dimensional, bi-periodic surface pattern, propagating on deep water, Hammack et al. [13] derived the two-dimensional vector nonlinear Schrödinger

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