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Mathematics and Computers in Simulation 105 (2014) 49–61

MATHEMATICS AND COMPUTERS IN SIMULATION

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Original Article

Real qualitative behavior of a fourth-order family of iterative methods by using the convergence plane☆

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Received 10 December 2013; received in revised form 7 March 2014; accepted 20 April 2014
Available online 20 May 2014

Abstract

The real dynamics of a family of fourth-order iterative methods is studied when it is applied on quadratic polynomials. A Scaling Theorem is obtained and the conjugacy classes are analyzed. The convergence plane is used to obtain the same kind of information as from the parameter space, and even more, in complex dynamics.
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Keywords: Real dynamics; Nonlinear problems; Convergence plane; Basins of attraction; Stability

1. Introduction

The application of iterative methods for solving nonlinear problems $f(x)=0$, with $f:\mathbb{R}\rightarrow\mathbb{R}$, gives rise to rational functions whose dynamics are not well-known. The simplest model is obtained when $f(x)$ is a quadratic polynomial and the iterative algorithm is Newton's scheme. This case has been widely studied under the view of complex dynamics (see, for instance [8,13]). The study of the dynamics of Newton's method has been extended to other point-to-point iterative schemes (see, for example [9,12,15,21]) and to multipoint iterative methods (see, for example [1,2,10,11,20,22]), for solving nonlinear equations. Nevertheless, the real dynamical analysis is not profusely studied, although some references can be found in the literature (see, for example [3,4,6,14,16,17]).

From the numerical point of view, the dynamical properties of the rational function associated with an iterative method give us important information about its stability and reliability. In most of mentioned papers, interesting dynamical planes, including periodical behavior and other anomalies, have been obtained. We are interested in the analysis of the role of the parameter in the stability of the family of iterative methods, which would allows us to select a particular one with good numerical properties.

☆ This research was supported by Ministerio de Ciencia y Tecnología MTM2011-28636-C02-{01,02}.
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http://dx.doi.org/10.1016/j.matcom.2014.04.006
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In this work, the family under study is the class of three-step fourth-order iterative methods for solving nonlinear systems $F(x) = 0$, introduced by the authors in [5], denoted by M4 and whose iterative expression is

$$\begin{aligned} y_k &= x_k - F'(x_k)^{-1} F(x_k), \\ z_k &= y_k - \frac{1}{\beta} F'(x_k)^{-1} F(y_k), \\ x_{k+1} &= z_k - F'(x_k)^{-1} \left(\left(\frac{2-1}{\beta - \beta} \right) F(y_k) + \beta F(z_k) \right), \end{aligned} \tag{1}$$

where β is an arbitrary complex parameter, $\beta \neq 0$. As the authors proved in [5], the corresponding scheme for $\beta = 1/5$ is the unique element of the family with order of convergence five. In this paper, we will analyze the real dynamical behavior of this family of methods applied to the nonlinear equation $f(x) = 0$. We will denote by $G_f(x, \beta)$ the fixed point operator associated to the class of methods (1) on a nonlinear real function $f(x)$, whose expression is

$$G_f(x, \beta) = x - \frac{1}{f(x)} \left[f(x) + f(y)(2-\beta) + \beta f\left(y - \frac{1}{\beta} \frac{f(y)}{f'(x)} \right) \right]$$

where $y = x - \frac{f(x)}{f'(x)}$.

The authors in [19] have studied the complex dynamics of the rational function associated to (1), analyzing the conjugacy classes. In it, some complex regions with rich dynamical behavior have been shown providing interesting elements of the family of iterative methods. In this paper, our aim is the study of the real dynamics of this family. Despite what might seem, some numerical results obtained in this paper allow us to conjecture that the real behavior is not included in the complex one. In fact, we will find stable real regions where attracting periodic orbits appear in the complex study, and vice versa.

The dynamical behavior associated to two rational operators (corresponding to a particular iterative method) conjugated by an affine map is qualitatively the same if a Scaling Theorem is satisfied.

Theorem 1. *Let $f(x)$ be an analytic function and let $A(x) = ax + b$ with $a \neq 0$ be an affine map. Let $g(x) = (f \circ A)(x)$. Then, $(A \circ G_g \circ A^{-1})(x, \beta) = G_f(x, \beta)$, that is, G_f and G_g are affine conjugated by A.*

Proof. Let us consider firstly

$$G_f(A(x), \beta) = T_f(A(x)) - \frac{(2 - (1/\beta) - \beta) f(N_f(A(x))) + \beta f(T_f(A(x)))}{f'(A(x))},$$

where T_f and N_f are the fixed-point operators of the first and second steps of (1), respectively.

As $g(x) = (f \circ A)(x)$, it is clear that $g'(x) = af'(A(x))$, $N_g(x) = x - (A(x) - N_f(A(x))/a)$ and also $g(N_g(x)) = f(N_f(A(x)))$. Then,

$$T_g(x) = x - \frac{1}{af'(A(x))} \left[f(A(x)) + \frac{1}{\beta} f(N_f(A(x))) \right]$$

and $g(T_g(x)) = f(T_f(A(x)))$.

Finally,

$$\begin{aligned} A(G_g(x, beta)) &= aG_g(x, \beta) + b \\ &= a \left[T_g(x) - \frac{(2 - (1/\beta) - \beta) g(N_g(x)) + \beta g(T_g(x))}{g'(x)} \right] + b \\ &= G_f(A(x), \beta) \end{aligned}$$

by simply substituting the previous calculations. □

The Scaling Theorem allows us to reduce the study of the dynamics of the iteration function G_f to the study of specific families of iterations of simpler maps. For example, any real quadratic polynomial $p(x) = c_2 x^2 + c_1 x + c_0$ with $c_2 \neq 0$ (we may assume that $c_2 = 1$) can be reduced to $x^2 + c$, by using an affine map. Then, we are going to analyze the following polynomials that correspond to the three different cases: $p_-(x) = x^2 - 1$ a particular case of two different real roots, $p_+(x) = x^2 + 1$ (a case with two complex roots) and $p_0(x) = x^2$, corresponding to multiple roots.

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