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Original Article

Dynamic complexities in a discrete predator–prey system with lower critical point for the prey ☆

Limin Zhang [a,b,*], Chaofeng Zhang [b], Min Zhao [c]

[a] College of Mathematics, Sichuan University, Chengdu, Sichuan 610064, China
[b] School of Mathematics and Finance-Economics, Sichuan University of Arts and Science, Dazhou, Sichuan 635000, China
[c] School of Life and Environmental Science, Wenzhou University, Wenzhou, Zhejiang 325027, China

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Abstract

In this paper, a discrete predator–prey system is proposed and analyzed. It is assumed that the prey population has a lower critical point, which is also referred to as extinction threshold. Such behavior has been reported for many flowering plants, many fishes, epidemiology, and so on. The existence and stability of nonnegative fixed points are explored. The conditions for the existence of flip bifurcation and Hopf bifurcation are obtained by using manifold theorem and bifurcation theory. Numerical simulations, including bifurcation diagrams, phase portraits and Maximum Lyapunov exponents, not only show the consistence with the theoretical analysis but also exhibit other complex dynamics and certain biological phenomena. Complex dynamics include quasi-periodicity, period-doubling bifurcations leading to chaos, chaotic bands with periodic windows, intermittent, supertransient, and so on. Simulations suggest that appropriate growth rate can stabilize the system, but the high growth rate may destabilize the stable system into more complex dynamics. As well, simulations suggest that the system is stable when the lower critical point parameter c is small, but when c increases beyond the critical values, the system changes from quasi-period to collapses. Furthermore, the simulated results are explained according to a biological point of view.

Keywords: Flip bifurcation; Hopf bifurcation; Lower critical point; Discrete predator–prey system; Dynamic complexities

1. Introduction

Population dynamics in ecology are generally governed by discrete-time and continuous-time systems. In recent years, the study of discrete-time biology systems has attracted extensive attentions [1,3,7,10,11,13–15,18,21–24,26, 29–36]. One important reason is that some natural populations have non-overlapping generations, thus discrete models are more realistic than continuous ones to study these species. Another reason is that people always study population changes by one year (mouth, week or day) in practice, thus, it is important and necessary to obtain discrete systems

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* Corresponding author. Tel.: +86 18281885162.
E-mail addresses: lmzhang2000@163.com, zlimin@yeah.net (L. Zhang), zbeyondiee@163.com (C. Zhang), zmcn@tom.com (M. Zhao).

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from continuous population dynamical models, by which one studies their dynamical properties [1,22,32]. Especially, using discrete models can provide more efficient computational models for numerical simulations and these results reveal richer dynamics of discrete models compared to continuous ones [1,7,15,22,24,30–36].

In nature, many species have their own lower critical points, which are also referred to as extinction threshold [4,5,24]. Once the population size is below its lower critical point, the species will die out. This phenomenon is also named in population dynamics as the negative competition effect [28]; in fisheries sciences, it is called critical depensation [12]; and in epidemiology, its analogous is the eradication threshold, the population level of susceptible individuals below which an infectious illness is eliminated from a population [5,8]. The lower critical point in biology may be caused by a variety of mechanisms operating in small populations, including the difficulty in finding mates, lessened defences against predators, and reduced foraging efficiency in social animal [19,27].

In this paper, we apply the forward Euler scheme to a predator–prey system with lower critical point for the prey and investigate this discrete-time dynamical system. The technique is also utilized by Zhang et al. [32], Liu and Xiao [22] and Agiza et al. [1].

First of all, we consider a Lotka–Volterra type predator–prey system [8,25]:

$$\begin{aligned}\dot{x}(t) &= r_0 x\left(1-\frac{x}{k}\right) - axy\\ \dot{y}(t) &= bxy - dy\end{aligned} \tag{1}$$

where $x(t)$and $y(t)$denote prey and predator densities respectively, all the parameters in system (1) are positive constants.

Considering the effect of population low critical point or extinction threshold, we modify system (1) and get the following system:

$$\begin{aligned}\dot{x}(t) &= r_0 x\left(1-\frac{x}{k}\right)(x-c) - axy\\ \dot{y}(t) &= bxy - dy\end{aligned} \tag{2}$$

where $x(t)$and $y(t)$ represent the population densities of prey and predator. In absence of predation, the prey grows with the term $r_0 x\left(1-\frac{x}{k}\right)(x-c)$, where r_0, where r_0 is intrinsic growth rate, k is the carrying capacity and $c>0$ is the lower critical point [6,2]. The lower critical point c is also referred to as extinction threshold [4,5,24], negative competition effect [28] or critical depensation [12], below which the prey growth rate is negative. Obviously, according to the biological meaning, the carrying capacity k is greater than the lower critical point c. The predator consumes the prey with a Holling-I functional response of ax and contributes to its growth with bx. The parameter d is the death rate of the predator.

For simplicity we rewrite the system above as

$$\begin{aligned}\dot{x}(t) &= rx(k-x)(x-c) - axy\\ \dot{y}(t) &= bxy - dy\end{aligned} \tag{3}$$

where $r=\frac{r_0}{k}$. When the carrying capacity k is fixed, the parameter r is a monotonically increasing function of r_0.

If biological population has non-overlapping generation or people study population changes by a cycle, as previously mentioned, it is necessary to obtain discrete systems from continuous population dynamical models. Then we apply the forward Euler scheme to system (3) and obtain the following discrete system:

$$\begin{aligned}x_{n+1} &= xn + rx_n(k-x_n)(x_n-c) - ax_n y_n\\ y_{n+1} &= yn + bx_n y_n - dy_n\end{aligned} \tag{4}$$

where the step size is one, x_n and y_n represent the population densities of prey and predator species respectively in generation n.

This paper is organized as follows. In Section 2, we study the existence and stability of fixed points of system (4). In Section 3, we give the sufficient conditions of existence for flip bifurcation and Hopf bifurcation. Numerical simulations are presented to verify the theoretical analysis and to exhibit other complex dynamics in Section 4. Finally, we give remarks to conclude this paper in Section 5.