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Original articles

Non-diminishing relative error of the predictor–corrector algorithm for certain fractional differential equations

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Highlights

- We find and analytically estimate the non-diminishing relative error of P–C algorithm for certain FEDs.
- We suggest a simple yet efficient strategy to reduce the non-diminishing relative error.
- The accuracy of P–C is significantly improved over the whole solution domain.

Abstract

The predictor–corrector (P–C) method applies linear interpolation technique to calculate Volterra integral equations equivalent to the considered fractional differential equations (FDEs). This paper reveals that, the relative error approaches a certain value but not infinitesimal even as the step size decreases to zero for certain FDEs. In these equations, the integrated function has a zero value and an infinite (or infinitesimal) slope at the origin. The interpolation technique is responsible for the non-diminishing relative error. Based on this analysis, we modify the P–C method by employing an alternative interpolation strategy to reduce the relative error. Numerical examples show the modified method can provide much more accurate approximations not only near the origin but also over the whole solution domain.
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Keywords: Fractional differential equation; Caputo derivative; Predictor–corrector method; Relative error

1. Introduction

Fractional differential equations (FDEs) invoke the increasing interest to model the real-world problems in many fields, such as, material science, biology, engineering, and others [1,14,23]. The reason partly consists in that fractional derivatives can better describe frequency dependence and memory properties of various phenomena [3,18,11,24].

Well-known, it is usually cumbersome to obtain an exact solution analytically for FDEs. Generally speaking, only approximate solutions can be obtained by analytical or semi-analytical solution techniques. There are several methods,

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which have been developed during the past decade, for instance, the homotopy analysis method [12,16], Admian's decomposition method [20,21] and variation iteration method [13,17], to mention a few.

Some of the above-mentioned methods are valid only for certain problems. In many occasions, numerical solution approaches are indispensable [5,7]. For example, researchers proposed some efficient explicit method containing finite difference method [25], spectral penalty methods [26] and block-by-block method [4], etc. Diethelm [9] proposed a famous implicit numerical scheme, called as the predictor–corrector (P–C) method, by extending the Adams-type method for FDEs. This method has been widely applied in various problems with fractional derivatives [2,15,22]. Deng [6] introduced short memory principle in the P–C method to improve computational efficiency. Later, Psihoyios and Simos [19] applied the trigonometrically fitting technique to modify the P–C scheme.

The P–C method is to implement linear interpolation techniques to Volterra integral equations. In the predictor process, the rectangle integral rule is adopted, and in the corrector procedure the trapezoidal integral rule is employed. We will restrict our attention to certain multi-term FDEs, in which the integrated function in the Volterra integral equation has a zero value and infinite (or infinitesimal) slope at the origin. Our major result is that the relative error does not diminish but instead approaches a non-zero value even if the discrete step decreases to 0. We elucidate this problem and give an analytical estimation on the non-diminishing relative error. More importantly, we will suggest a strategy to eliminate the non-diminishing error and hence significantly improve the accuracy of the P–C algorithm. Finally, numerical examples are presented to validate the modified P–C method.

2. Predictor–corrector method for multi-term FDEs

Simulations of multi-term FDEs have also attracted the curiosities of many researchers. Diethelm [8] originally employed the P–C scheme to multi-term FDEs by transforming them into a system of equations of the same fractional order which is the greatest common divisor of multi-orders. A requirement for this approach is that the multi-orders need to be reducible such that there is a common divisor. EI-Mesiry et al. [10] and Edwards et al. [9] suggested two different discrete strategies, respectively, so that the fractional order can take arbitrary value. It will help us to reduce the dimension of the equations for many cases.

In order to implement the P–C algorithm, many mathematical and engineering problems containing multiple fractional derivatives are modeled by a system of equations as

$$\begin{aligned} \frac{d^{\beta_1} y_1}{dt^{\beta_1}} &= f_1(t, y_1, y_2, \ldots, y_n) \\ \frac{d^{\beta_2} y_2}{dt^{\beta_2}} &= f_2(t, y_1, y_2, \ldots, y_n) \\ &\vdots \qquad\qquad \vdots \\ \frac{d^{\beta_n} y_n}{dt^{\beta_n}} &= f_n(t, y_1, y_2, \ldots, y_n) \end{aligned} \tag{1}$$

subject to the initial condition $y_i(0) = y_{i0}$. Here, $\frac{d^{\beta_i} y_i}{dt^{\beta_i}}$ denotes the Caputo derivative of order $\beta_i \in (0, 1]$ defined as

$$\frac{d^{\gamma} x}{dt^{\gamma}} = J^{m-\gamma} \frac{d^m y}{dt^m} \tag{2}$$

where $m := \lceil \gamma \rceil$ is the value γ rounded up to the nearest integer, J is Riemann–Liouville integral operator determined by

$$J^{\eta} z(t) = \frac{1}{\Gamma(\eta)} \int_0^t (t - v)^{\eta - 1} z(v) dv. \tag{3}$$

Note that $\eta = m - \gamma > 0$. The initial value problem (1) can be transformed into the Volterra integral equations

$$y_i(t) = y_{i,0}^{+} \frac{1}{\Gamma(\beta_i)} \int_0^t (t - v)^{\beta_i - 1} f_i(v, y(v)) dv \tag{4}$$

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