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A quadratic temporal finite element method for linear elastic structural dynamics

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Abstract

Recently, in order to overcome the key shortcoming in Hamilton's principle, the extended framework of Hamilton's principle was formulated. This new variational framework provides a sound base to develop novel temporal finite element methods with the proper use of initial conditions to the strong form. As its initial and practical application in structural dynamic analysis, this paper presents a quadratic temporal finite element method for linear elastic multi-degrees-of-freedom systems. Numerical features of the developed method are analytically and numerically investigated with comparison to the currently dominant method; the results highlight the improved performance of the proposed method.
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1. Introduction

In current practice [10–12,15,17,19,23,31,35,37,44], linear or nonlinear analysis of dynamic systems depends on the time integration scheme that essentially requires separate discretization of space and time. That is, with conventional finite element methods in statics, we spatially model a dynamic system first, then, introduce a time integration technique to have a system of algebraic equations at each time-step (a semi-discrete method). However, any semi-discrete method cannot be a unified space–time finite element method, because it does not originate from variational framework of dynamics.

For a long time, Hamilton's principle [28,29] has provided a fundamental basis of theoretical dynamics. However, it has two main difficulties. First, variations are not compatible to the specified initial conditions of the strong form, and second, irreversible process cannot be incorporated in the purely variational manner. The first difficulty associates with the restrictions on the function variations. In Hamilton's principle, the variation terms vanish at both the starting and ending of time interval, which means that functions should be known at these two time instants. However, in typical dynamic problems, one cannot know how the dynamic system evolves at the end of

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the time interval. Usually, this is the main objective of the analysis, which means there may be serious philosophical inconsistency in the framework of Hamilton's principle. The second difficulty relates to the inability to integrate the irreversible phenomena. Hamilton's principle itself only applies to conservative systems. With a separate quadratic dissipation function called Rayleigh dissipation [38], irreversible phenomena can be brought into the framework of Hamilton's principle. While this approach is not completely satisfactory in a strict mathematical sense (since taking variation requires *ad hoc* rules), it provides an appropriate framework to accommodate mathematically modeled non-conservative systems. Representative examples using this approach within mixed Lagrangian formulation can be found in some references [5,6,32,41,42].

In order to overcome both difficulties in Hamilton's principle, Gurtin [24–26] introduce temporal convolution functional for viscoelasticity and elastodynamics, while Tonti [45] provides an insightful assessment of variational methods for dynamic problems and advocates the use of the convolutional bilinear form. Following the ideas of Tonti and Gurtin, Oden and Reddy [36] extend the formulation to initial and boundary value problems in mechanics, especially for Hellinger–Reissner type mixed principles. However, the approach by Gurtin changes the initial value problem to equivalent integral equation, and in return, it never can recover the original strong form. Also, the approach by Tonti cannot properly assess non-homogeneous initial values and physical meaning of momentum equation. Riewer [39,40] adopts the fractional calculus to permit the development of a single scalar function for dissipative dynamic systems, and many other researchers proposed similar approaches through the fractional integro-differentiation [1–4,7,8,20–22]. However, none of these papers describe analytical or numerical demonstration validating their approach to the most fundamental case, a classical single-degree-of-freedom (SDOF) linear mass–spring–damper system.

Recently, a new variational framework, named the Extended framework of Hamilton's Principle (EHP, [30]), was established for J_2-viscoplastic and elastic continua dynamics to account proper initial conditions in Hamilton's principle. This framework utilizes mixed Lagrangian formulation and sequentially assigning process to take the proper initial conditions into account within Hamilton's principle. Theoretically, this extension framework holds, since (i) it only considers unique dynamic evolution cases of the system by additional terms in the action variation and (ii) the true evolving trajectory of the system is found through sequentially identifying the given initial values among these unique cases. EHP does not provide a complete variational framework of dynamics, because it requires both (i) Rayleigh's dissipation for dissipative systems, and (ii) external specification of initial conditions in the variation of action. However, it provides a sound base to develop novel unified space–time finite element methods.

In this paper, as its initial and practical uses for structural dynamic analysis, this new variational framework is applied to a linear elastic multi-degrees-of-freedom (MDOF) system and corresponding weak formulation is discretized in time domain with the adoption of quadratic temporal shape functions. Through the analytical investigation on a conservative system, it is shown that the developed method provides a symplectic, time-reversible, volume-preserving, unconditionally stable and energy-conservation algorithm, along with less period elongation property compared to the currently dominant methods. Also, an example of three-story shear building is presented in order to demonstrate the performance of the proposed method.

2. Extended framework of Hamilton's principle (EHP)

Previously, with (i) a sequential viewpoint for Hamilton's principle and (ii) a consequent correction for problematic steps in the framework of Hamilton's principle, the EHP is suggested for elastic and J_2 viscoplastic continuum dynamics in mixed Lagrangian formulation [30]. The new variational framework is formulated to resolve the end-point constraint issue in Hamilton's principle, and it can also apply to non-conservative system with the Rayleigh's dissipation. In this section, this EHP is applied to both SDOF and MDOF systems for further development of a quadratic temporal finite element method.

2.1. Weak form of SDOF oscillator

With mass m, damping coefficient c, the applied force $\bar{f}(t)$ with time t, and stiffness $k = 1/a$ with a representing the flexibility in Fig. 1, the governing differential equations of the forced-damped system can be written in terms of the displacement of the mass $u(t)$ and the impulse of the internal spring force $J(t)$ as

$$m\ddot{u} + c\,\dot{u} + \dot{J} - \bar{f} = 0; \quad a\ddot{J} - \dot{u} = 0 \tag{1}$$

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