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Isogeometric analysis: An overview and computer implementation aspects

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Abstract

Isogeometric analysis (IGA) represents a recently developed technology in computational mechanics that offers the possibility of integrating methods for analysis and Computer Aided Design (CAD) into a single, unified process. The implications to practical engineering design scenarios are profound, since the time taken from design to analysis is greatly reduced, leading to dramatic gains in efficiency. In this manuscript, through a self-contained Matlab® implementation, we present an introduction to IGA applied to simple analysis problems and the related computer implementation aspects. Furthermore, implementation of the extended IGA which incorporates enrichment functions through the partition of unity method (PUM) is also presented, where several examples for both two-dimensional and three-dimensional fracture are illustrated. We also describe the use of IGA in the context of strong-form (collocation) formulations, which has been an area of research interest due to the potential for significant efficiency gains offered by these methods. The code which accompanies the present paper can be applied to one, two and three-dimensional problems for linear elasticity, linear elastic fracture mechanics, structural mechanics (beams/plates/shells including large displacements and rotations) and Poisson problems with or without enrichment. The Bézier extraction concept that allows the FE analysis to be performed efficiently on T-spline geometries is also incorporated. The article includes a summary of recent trends and developments within the field of IGA.

Keywords: Isogeometric analysis; NURBS; Finite elements; CAD; Isogeometric collocation

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1. Introduction

1.1. Underlying concepts of isogeometric analysis

The predominant technology that is used by Computer Aided Design (CAD) to represent complex geometries is the Non-Uniform Rational B-spline (NURBS). This allows certain geometries that can only be approximated by polynomial functions to be represented exactly. Examples of this include conic sections and objects such as cylinders, spheres, etc. which are essential in design. There is a vast array of literature focused on NURBS (e.g. [96,104]) and as a result of several decades of research, many efficient computer algorithms exist for their fast evaluation and refinement. The key concept outlined by Hughes et al. [55] was to employ NURBS not only as a geometry discretization technology, but also as a discretization tool for analysis, attributing such methods to the field of 'Isogeometric Analysis' (IGA). Since this seminal paper, a monograph dedicated entirely to IGA has been published [27] and applications can now be found in several fields including structural mechanics, solid mechanics, fluid mechanics and contact mechanics.

We give in this section an overview of some of these recent developments while outlining the benefits and present shortcomings of IGA. It should be emphasized that the idea of using CAD technologies in finite elements (FEs) dates back at least to [65,64] where B-splines were used as shape functions in Finite Element Method (FEM). In addition, similar methods which adopt subdivision surfaces have been used to model shells [24]. We also review some of the recent attempts at simplifying the CAD–FEA (FEA stands for Finite Element Analysis) integration by separating boundary and domain discretizations.

1.2. Applications

In contact formulations using conventional geometry discretizations, the presence of faceted surfaces can lead to jumps and oscillations in traction responses unless very fine meshes are used. The benefits of using NURBS over such an approach are evident, since smooth contact surfaces are obtained, leading to more physically accurate contact stresses. Recent work in this area includes [137,61,138,74,80,136].

IGA has also shown advantages over traditional approaches in the context of optimization problems, [145,77,99,98] where the tight coupling with CAD models offers an extremely attractive approach for industrial applications. Another attractive class of methods include those that require only a boundary discretization, creating a truly direct coupling with CAD. Isogeometric boundary element methods for elastostatic analysis were presented in [127,118], demonstrating that mesh generation can be completely circumvented by using CAD discretizations for analysis.

Shell and plate problems are another field where IGA has demonstrated compelling benefits over conventional approaches [13,68,14,11,140,40,16]. The smoothness of the NURBS basis functions allows for a straightforward construction of plate/shell elements. Particularly for thin shells, rotation-free formulations can be easily constructed [68,67]. Note that for multi-patch NURBS surfaces, rotation-free IGA elements require special treatment at patch boundaries where the basis functions are found to be C^0 continuous. Furthermore, isogeometric plate/shell elements exhibit much less pronounced shear-locking compared to standard FE plate/shell elements. Elements with smooth boundaries such as circular and cylindrical elements were successfully constructed using the IGA concept [75,76].

The smoothness of NURBS basis functions is attractive for analysis of fluids [51,94,7] and for fluid–structure interaction problems [9,10]. In addition, due to the ease of constructing high order continuous basis functions, IGA has been used with great success in solving PDEs that incorporate fourth order (or higher) derivatives of the field variable such as the Hill–Cahnard equation [50], explicit gradient damage models [142] and gradient elasticity [48]. The high order NURBS basis has also found potential applications in the Kohn–Sham equation for electronic structure modeling of semiconducting materials [79].

NURBS provide advantageous properties for structural vibration problems [28,56,139,147] where k-refinement (unique to IGA) has been shown to provide more robust and accurate frequency spectra than typical higher-order FE p-methods. Particularly, the optical branches of frequency spectra, which have been identified as contributors to Gibbs phenomena in wave propagation problems (and the cause of rapid degradation of higher modes in the p-version of FEM), are eliminated. However when lumped mass matrices were used, the accuracy is limited to second order for any basis order. High order isogeometric lumped mass matrices are not yet available. The mathematical properties of IGA were studied in detail by [46].