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Original Article

Local interpolation schemes for landmark-based image registration: A comparison

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Abstract

In this paper we focus, from a mathematical point of view, on properties and performances of some local interpolation schemes for landmark-based image registration. Precisely, we consider modified Shepard's interpolants, Wendland's functions, and Lobachevsky splines. They are quite unlike each other, but all of them are compactly supported and enjoy interesting theoretical and computational properties. In particular, we point out some unusual forms of the considered functions. Finally, detailed numerical comparisons are given, considering also Gaussians and thin plate splines, which are really globally supported but widely used in applications.

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1. Introduction

Image registration is an important challenging topic in image processing. It consists mainly in finding a suitable transformation between two images (or image data), called *source* and *target images*, taken either at different times or from different sensors or viewpoints. The scope is to determine a transformation such that the transformed version of the source image is similar to the target one. There is a large number of applications demanding image registration, including astronomy, biology, computer vision, genetics, physics, medicine, robotics, to name a few. For an overview, see e.g. [20,21,25,26,32,34–36,40,42,48] and references therein. In medicine, for example, registration is required for combining different modalities (X-ray, computer tomography (CT), magnetic resonance imaging (MRI) and positron emission tomography (PET) images, for instance), monitoring of diseases, treatment validation, comparison of the patient's data with anatomical atlases, and radiation therapy. In particular, the *landmark-based image registration* process is based on two finite sets of landmarks, i.e. sparse data points located on images, usually not uniformly distributed, where each landmark of the source image has to be mapped onto the corresponding landmark of the target image (see [34,35,40]). Now, in order to give a more formal idea, we consider the sets $\mathcal{S}_N = \{\mathbf{x}_j \in \mathbb{R}^m,\ j = 1, 2, \ldots, N\}$

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and $\mathcal{T}_N = \{\mathbf{t}_j \in \mathbb{R}^m,\ j = 1, 2, \ldots, N\}$ each containing N point-landmarks in the source and target images, respectively. Thus, the registration problem involves a transformation $\mathbf{F} : \mathbb{R}^m \to \mathbb{R}^m$, such that

$$\mathbf{F}(\mathbf{x}_j) = \mathbf{t}_j, \quad j = 1, 2, \ldots, N,$$

where each coordinate F_k of the transformation function $\mathbf{F} = (F_1, F_2, \ldots, F_m)^T$ is separately calculated, that is, the interpolation problem involving $F_k : \mathbb{R}^m \to \mathbb{R}$ is solved for $k = 1, 2, \ldots, m$, with the corresponding conditions $F_k(\mathbf{x}_j) = t_{jk}$, $j = 1, 2, \ldots, N$. This problem can be formulated in the context of multivariate scattered data interpolation, and solved by different techniques, among which radial basis functions (RBFs) play a preminent role (see, e.g., [10,28,46]). The use of RBF transformations, in particular of the thin plate splines, for point-based image registration was first proposed by Bookstein [8], and it is still common (see [37] and the software package MIPAV [33]). A number of authors have investigated the most popular radial basis function transformations in the image registration context: thin plate spline [7,31], multiquadric [30,41], inverse multiquadric [41], and Gaussian transformations [7]. A more specific application which involves registration and includes imaging techniques, such as computer tomography and magnetic resonance imaging, can be found in [37,38].

Since using globally supported RBFs, as for example the Gaussians, a single landmark pair change may influence the whole registration result, in the last two decades several methods have been presented to circumvent this disadvantage, such as weighted least squares and weighted mean methods (WLSM and WMM, respectively) [24], compactly supported radial basis functions (CSRBFs), especially Wendland's and Gneiting's functions [14,15,23], and elastic body splines (EBSs) [29].

A certain number of papers have been dedicated to recall and compare these methods for nonrigid image registration: main contributions, advantages and drawbacks of radial basis functions, compactly supported radial basis functions and elastic body splines are mentioned in [48]; thin plate splines, multiquadrics, piecewise linear and weighted mean transformations are explored and their performances are compared in [47]; finally, radial basis functions, Wendland's functions and elastic body splines are reviewed, together with B-splines and wavelets, in [27].

Several authors have shown the superiority of local registration methods over the global ones in some situations, for instance, in medical imaging and in airborne imaging. In fact, a global mapping cannot properly handle images locally deformed. For this reason, more recently, local methods, already known in interpolation theory, have been proposed in landmark-based image registration: the modified Shepard's method (also known as the inverse distance weighted method (IDWM)) [11,12], and Lobachevsky spline method [1,3]. These interpolation techniques, giving rise to local mappings, handle well images locally deformed. Moreover, they are in general stable and the computational effort to determine transformations is low and, therefore, a large number of landmarks can be used.

In this paper we focus, from a mathematical point of view, on properties and performances of some local interpolation schemes for landmark-based image registration. Precisely, we consider modified Shepard's interpolants, Wendland's functions, and Lobachevsky splines. They are quite unlike each other, but all of them are compactly supported and enjoy interesting theoretical and computational properties (see [46,13]). In particular, we point out some unusual forms of the considered functions. Moreover, referring to Wendland's functions, we consider for the first time in this context, as far as we know, compactly supported interpolants given by products of univariate Wendland's functions [16]. All these methods are also compared with Gaussians and thin plate splines, which are globally supported but are still among the most widely used methods in applications.

Numerical experiments point out differences in accuracy and smoothness of the considered methods. The comparison can be useful to users in the choice of the appropriate transformation for their scopes. Moreover, since some schemes need parameters, our numerical tests might be of interest in the choice of them.

The paper is organized as follows. Section 2 introduces some preliminaries: the landmark-based registration problem and the solvability of the associated interpolation problem. In Section 3 we briefly recall radial basis functions, like Gaussians, multiquadrics, inverse multiquadrics and thin plate splines to construct globally supported transformations. Section 4 is devoted to describe local transformations, given by the modified Shepard's formula which uses RBFs as local approximants. In Section 5 Wendland's functions are presented to define compactly supported transformations, whereas in Section 6 we focus on Lobachevsky splines which define again compactly supported transformations. Finally, Section 7 contains several numerical results obtained in some test and real-life examples: special emphasis is devoted to comparing accuracy of local interpolation schemes and to determining optimal values of parameters.