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Original article

There are simple and robust refinements (almost) as good as Delaunay

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Abstract

A new edge-based partition for triangle meshes is presented, the Seven Triangle Quasi-Delaunay partition (7T-QD). The proposed partition joins together ideas of the Seven Triangle Longest-Edge partition (7T-LE), and the classical criteria for constructing Delaunay meshes. The new partition performs similarly compared to the Delaunay triangulation (7T-D) with the benefit of being more robust and with a cheaper cost in computation. It will be proved that in most of the cases the 7T-QD is equal to the 7T-D. In addition, numerical tests will show that the difference on the minimum angle obtained by the 7T-QD and by the 7T-D is negligible.

Keywords: Longest-edge; Edge-refinement; Delaunay

1. Introduction

Longest edge-based refinement has become popular in last decade in the context of mesh refinement [9,11,14]. A well accepted acronym used to name this class of mesh subdivision is nT-LE, where n is the number of new triangles (T) produced after a single subdivision and LE stands for longest edge. So, we found in the literature well studied longest-edge partition as 2T-LE, 3T-LE, 4T-LE and 7T-LE, see [14,15,9] and the references therein. It should be noted that the iterative application of these partitions yields good-quality meshes, in the sense that they do not degenerate. Additionally, longest edge refinements have the advantage of its propagation, i.e., if we subdivide a triangle, we know how to subdivide its adjacent triangles in order to obtain a conforming triangulation.

Of course, if we add some points to a triangle, and we want to obtain a subdivision with the best quality (in the sense that we want to maximize the minimum angle), the optimal solution is the Delaunay triangulation (see, for example [1,2]). Mesh generation algorithms based on Delaunay refinement have been effective tools both in theory and in practice in the last 20 years [3,19,5]. The first provably good Delaunay refinement algorithm is due to Chew [4]. Much attention has received this class of algorithms afterwards, in particular thanks to authors like Ruppert [16] and Shewchuk [17,18] among others. Longest-edge based algorithms have been used together with Delaunay triangulation for the quality triangulation problem by Rivara and co-workers [8,13].

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Unfortunately, Delaunay refinement algorithms present some disadvantages from a practical point of view: on one hand they require a considerable amount of computation, and, on the other hand, they are not robust dealing with point sets that are not in general position (a point set is in general position if no three of them lie on the same straight line and no four lie on the same circle). It should be noted here that longest edge refinements always present point sets that are not in general position (we will discuss this topic in the next section). In order to solve these inconveniences, some refinements that perform only comparisons of distances have been presented (see [14,15,9]. Of course, to measure a distance between pairs of points is easier and far more robust than compare angles or doing circumcircle tests, and so in the mentioned works (and in many others) the authors avoid the theoretical advantages of Delaunay triangulations for the sake of the simplicity and robustness. In this way, many methods appear as those mentioned above [14,15,9].

In this work, we try to obtain the best of both worlds, we propose a longest edge refinement (the 7T-QD refinement) that can be obtained performing only lengths comparisons, and that in most of the cases, actually more than in 97% of the triangles, coincides with the Delaunay triangulation. Even in the cases that the 7T-QD is not a Delaunay triangulation, we are not far from that optimal refinement in the following ways: five out of seven of the triangles that are obtained with the 7T-QD refinement from a original triangle are Delaunay triangles, and the minimum angle of the other two triangles are, in the worst case, only a 20% worse than the minimum angle of the Delaunay refinement, but the refinement presented is better if we measure the average of the minimum angle of the two triangles.

The structure of this paper is as follows: Section 2 gives a short background of the class of refinement methods treated in this paper, Section 3 introduces the Seven Quasi-Delaunay partition for triangles and gives a comprehensive comparison with the pure Delaunay triangulation and the Seven Triangles Longest-Edge partition. In Section 4, we provide a numerical study considering the min angle that stress the quality of 7T-QD. Finally some useful conclusions regarding the new introduced partition are offered.

2. Triangulation with n aligned points

By locating midpoints on the edges of the triangle we can compute some quality triangulations in the plane, see [9] and the references therein. This can be viewed as triangulation with n aligned points. One of the interesting points of such family of triangulations is the low cost for obtaining the subdivision.

In last decade, subdivision methods inserting one point per edge – based on the longest edge or not – have been sufficiently explored. Some of these partitions are: "red–green", longest edge bisection (2T-LE), and four triangles longest edge bisection (4T-LE). Less attention has been given to subdivisions based on the insertion of two points per edge.

Lastly, the Seven Triangle Longest-Edge partition (7T-LE) has been presented in [9]. The 7T-LE partition of a triangle t is obtained by putting two equally spaced points per edge. After cutting off three triangles at the corners, the remaining hexagon is subdivided further by joining each point of the longest-edge of t to the base points of the opposite sub-triangle. Finally, the interior quadrangle is subdivided into two sub-triangles by the shortest diagonal, see Fig. 1.

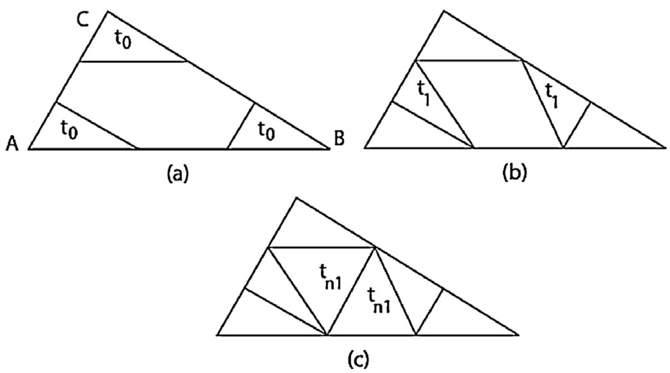

Fig. 1. Scheme for the 7T-LE partition of triangle t_0 and new class of triangles generated, t_1 and t_{n1}.