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Original Article

# Efficient geometric reconstruction of complex geological structures

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**Abstract**

Complex geological structures pose a challenge to domain discretization. Indeed data are normally given as a set of intersecting surfaces, sometimes with incomplete data, from which one has to identify the computational domain to build a mesh suited for numerical simulations. In this paper, we describe a set of tools which have been developed for this purpose. Specialized data structures have been developed to efficiently identify intersections of triangulated surfaces and to conformally include these intersections in the starting meshes, while improving the mesh quality. Then, an effective algorithm has been implemented to detect the different subregions forming the computational domain; this algorithm has been properly enhanced to take into account the specific characteristics involved in the simulation of geological basins.


*Keywords:* Triangulated surface intersection; Search data structures; Geological model reconstruction

## 1. Introduction

In many applications the geometry of the computational domain comes from different kind of tomography processes, acoustic, seismic and X-ray imaging being only few examples. In fact, these processes are usually able to give information on the geometry of the surfaces that bound the different parts of the domain of interest. Specific procedures are then necessary to convert this information to a geometrical description suitable for the successive analysis, which typically involve the numerical solution of partial differential equations.

For example, data from seismic imaging offer a description of the horizons, i.e., a set of three dimensional surfaces that represent the deposition of different kind of sediments (see Fig. 1, left). Moving from these data, we have to get a volume discretization representing the whole sedimentary basin (see Fig. 1, right).

We are indeed interested in the geometrical reconstruction of a sedimentary basin moving from the description of horizons, usually given in the form of a triangulated surfaces. There is a sequence of operations that are required to finally obtain a computational mesh representing the whole sedimentary basin on which one can carry out numerical simulations. We will focus on the following procedures:

(a) identification of surface intersection;

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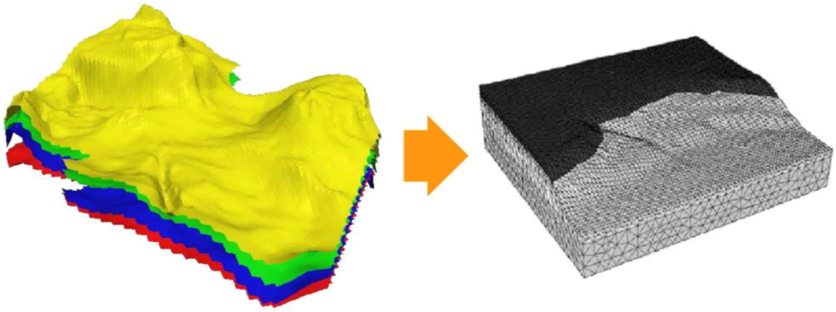

Fig. 1. Surfaces that represent the horizons (left); the volume of interest (right).

(b) detection of regions enclosed by this intersection and generation of a conformal mesh which includes such intersection;
(c) improvement of the mesh quality.

In geological applications, the data sets that represent the different horizons have large size; so an efficient implementation of the previous algorithms, especially of the surface intersection, becomes important. We have devised proper data structures based on the combination of an Alternated Binary Tree with a Structured Space discretization, to address this issue and to obtain an efficient code.

The intersections partition the surface into regions which have to be identified, since they represent the boundary of different portions of the computational domain, typically with different characteristics. We propose here a simple algorithm for the automatic recognition of those regions. In this process, we also re-mesh the surface so that the new triangulation is conformal with the intersections. This process is complemented by a mesh enhancement procedure to avoid badly shaped triangles. Finally, we are able to identify the different portions of the three dimensional geometry automatically.

Another aspect peculiar to the target application is that data are often incomplete. We thus need to reconstruct the missing parts. We present here two possible strategies that we have successfully implemented. The algorithms have been implemented using the C++ language to exploit generic and object oriented programming techniques.

The paper is organized as follows. Section 2 deals with the problem of finding the intersection between two triangulated surfaces and details the data structures that have been used for this purpose. The localization of the intersection is a prerequisite for the procedure we illustrate in Section 3, where we deal with the automatic detection of the different regions composing our 3D model. The surface mesh provided by the original data set and successively modified by the intersection finding procedure is often not suitable for computations. The quality of the elements is usually poor. In Section 4, we outline the mesh improvement strategies we have consequently adopted. In Section 5, we describe some techniques specific for the application at hand, namely the completion of defective geometrical data, the treatment of the so-called hard and soft horizons, and the construction of sub-volumes. Finally, Section 6 provides some more quantitative results about the algorithms proposed in Sections 2–3. Some conclusions are drawn in the last section.

## 2. Intersection of horizons and faults

The goal of this section is to describe an efficient method to detect the intersection between horizons and faults in a sedimentary basin. The latter are described by triangulated surfaces.

For the sake of simplicity, we focus on a single intersection between a fault and a horizon, even though realistic geological configurations include multiple intersections (see Fig. 2 for two examples). We can consequently formalize the detection of such an intersection via the simple geometric configuration illustrated in Fig. 3. Here, the blue surface $\mathcal{S}^B$ represents a fault which intersects the horizon given by the red surface $\mathcal{S}^R$. In particular, since we deal with two triangulated surfaces, we are led to identify the yellow piecewise linear curve $\Gamma$ in Fig. 3(right), i.e., to search the intersection between couples of non-coplanar triangles.