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Mathematics and Computers in Simulation 81 (2011) 2595–2607

MATHEMATICS AND COMPUTERS IN SIMULATION

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Original article

Center conditions and bifurcation of limit cycles at three-order nilpotent critical point in a septic Lyapunov system ☆

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Received 4 July 2010; received in revised form 23 March 2011; accepted 2 May 2011
Available online 18 May 2011

Abstract

In this paper, center conditions and bifurcation of limit cycles at the nilpotent critical point in a class of septic polynomial differential systems are investigated. With the help of computer algebra system MATHEMATICA, the first 13 quasi-Lyapunov constants are deduced. As a result, sufficient and necessary conditions in order to have a center are obtained. The result that there exist 13 small amplitude limit cycles created from the three order nilpotent critical point is also proved. Henceforth we give a lower bound of cyclicity of three-order nilpotent critical point for septic Lyapunov systems.
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MSC: 34C05; 34C07

Keywords: Three-order nilpotent critical point; Center-focus problem; Bifurcation of limit cycles; Quasi-Lyapunov constant

1. Introduction

The nilpotent center problem was investigated by Moussu [16] and Stróżyna [17]. Nevertheless, given an analytic system with a monodromic point, it is still very difficult to detect if it is a focus or a center, even in the case of a concrete polynomial systems. In this paper, we consider an autonomous planar ordinary differential equation having a three-order nilpotent critical point with the form

$$\begin{aligned}\frac{dx}{dt} &= y + y^2 - x^2y + a_{12}xy^2 + a_{50}x^5 + a_{05}y^5 + a_{06}y^6 + 6b_{06}xy^5 \\ &\quad + a_{33}x^3y^3 + \frac{5}{2}b_{15}x^2y^4 + a_{42}x^4y^2 + a_{07}y^7 + a_{52}x^5y^2 + a_{34}x^3y^4, \\ \frac{dy}{dt} &= -2x^3 - a_{12}x^2y + b_{03}y^3 - 5a_{50}x^4y + b_{60}x^6 - b_{06}y^6 - \frac{3}{4}a_{33}x^2y^4 \\ &\quad - b_{15}xy^5 - \frac{4}{3}a_{42}x^3y^3 + b_{34}x^3y^4,\end{aligned} \tag{1.1}$$

☆ This research is partially supported by the National Nature Science Foundation of China (11071222).

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doi:10.1016/j.matcom.2011.05.001

where

$$b_{34} = \frac{27679281609501193}{3330432589979136}a_{52}^2, \quad a_{42} = -\frac{982386995017}{384124133250}.$$

In some suitable coordinates, the Lyapunov system with the origin as a nilpotent critical point can be written as

$$\begin{aligned} \frac{dx}{dt} &= y + \sum_{i+j=2}^{\infty} a_{ij}x^i y^j = X(x, y), \\ \frac{dy}{dt} &= \sum_{i+j=2}^{\infty} b_{ij}x^i y^j = Y(x, y). \end{aligned} \tag{1.2}$$

Suppose that the function $y=y(x)$ satisfies $X(x, y)=0$, $y(0)=0$. Lyapunov proved (see for instance [3]) that the origin of system (1.2) is a monodromic critical point (i.e., a center or a focus) if and only if

$$\begin{aligned} &Y(x, y(x)) = \alpha x^{2n+1} + o(x^{2n+1}), \quad \alpha < 0, \\ &\left[\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial x}\right]_{y=y(x)} = \beta x^n + o(x^n), \\ &\beta^2 + 4(n+1)\alpha < 0, \end{aligned} \tag{1.3}$$

where n is a positive integer. The monodromy problem for a general nilpotent singularity was solved in [4] and the center problem in [16], see also in [17]. As far as we know, there are essentially three differential ways of obtaining Lyapunov constants for a monodromic nondegenerate singular point by using normal form theory [9], by computing the Poincaré return map [6] or by using Lyapunov functions [18]. These three ways have been also used to study the center-focus problem of nilpotent critical points. In [1] the monodromy and stability for nilpotent critical points with the method of computing the Poincaré return map is investigated. In [8] the local analytic integrability of nilpotent centers is studied by using Lyapunov functions. In [16] Moussu investigated the center-focus problem of nilpotent critical points with the normal form theory.

In [19], Takens proved that system (1.2) can be formally transformed into a generalized Liénard system. Furthermore, in [2] it is proved that the generalized Liénard system could be simplified even more by a reparametrization of the time. At the same time, Giacomini, et al. in [12,13] proved that the analytic nilpotent system with a center can be expressed as limit of non-degenerate system with a center. Therefore, any nilpotent center can be detected using the same methods that for a non-degenerate center, for instance the Poincaré-Lyapunov method can be used to find the nilpotent centers.

There are very few results known for concrete differential systems with monodromic nilpotent critical points. Gasull and Torregrosa in [10] have generalized the scheme of computation of Lyapunov constants for systems of the form

$$\begin{aligned} \dot{x} &= y + \sum_{k\geq n+1} F_k(x, y), \\ \dot{y} &= -x^{2n-1} + \sum_{k\geq 2n} G_k(x, y), \end{aligned} \tag{1.4}$$

where F_k and G_k are $(1, n)$-quasi-homogeneous functions of degree k. Chavarriga, García, and Giné investigated the integrability of centers perturbed by (p, q)-quasi-homogeneous polynomials in [7].

For a given family of polynomial differential equations in general the number of Lyapunov constants needed to solve the center-focus problem is also related with the so-called cyclicity of the point, i.e. the number of limit cycles that appear from it by small perturbations of the coefficients of the given differential equation inside the family considered (see [11] for cases where this relation does not exist for the case of nondegenerate centers). let $N(n)$ be the maximum possible number of limit cycles bifurcating from nilpotent critical points for analytic vector fields of degree n. It was found that $N(3) \geq 2$, $N(5) \geq 5$, $N(7) \geq 9$ in [5], $N(3) \geq 3$, $N(5) \geq 5$ in [1], and for a family of Kukles system with 6 parameters $N(3) \geq 3$ in [2]. At Recently, Liu and Li proved that $N(3) \geq 8$ in [15]. In this paper, employing the inverse integral factor method introduced in [14], see also in [15], we will prove $N(7) \geq 13$. To the best of our knowledge, our result on the lower bounds of cyclicity of three-order nilpotent critical points for septic systems is new.

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