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Mathematics and Computers in Simulation 82 (2012) 1788–1798

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Original article

Preconditioning of the coarse problem in the method of balanced domain decomposition by constraints

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Received 20 October 2009; received in revised form 27 March 2011; accepted 13 June 2011
Available online 8 September 2011

Abstract

The method of balanced domain decomposition by constraints is an iterative algorithm for numerical solution of partial differential equations which exploits a non-overlapping partition of a domain. As an essential part of each step, restricted problems are solved on every subdomain and a certain coarse grid solution is found. In this paper we present a new strategy of preconditioning of the coarse problem. This is based on the algebraic multilevel preconditioning technique. We present numerical estimates of constants defining the condition numbers of the preconditioned coarse problems for several two- and three-dimensional elliptic equations.
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Keywords: Domain decomposition; Multilevel methods; Hierarchical methods; Preconditioning; Strengthened CBS inequality

1. Introduction

Decomposition of domains and multilevel methods together with their parallel implementations have become a powerful tool for solving partial differential equations discretized by the finite element (FE) method. In this paper we consider one of these strategies applied to the second order elliptic equations. It is the method of balanced domain decomposition by constraints (BDDC), see for example [5,8–10,12,13]. A domain where the problem is defined is partitioned into non-overlapping subdomains. As the main part of each iteration, restricted problems on every subdomain and a certain coarse grid problem have to be solved. Base functions for the coarse problem have minimal energy on every subdomain and satisfy certain continuity conditions on the interfaces of subdomains, the so called coarse degrees of freedom (DOF). The basic notation and description of the BDDC algorithm are shown in Section 2.

As the coarse DOFs, some nodal values on interfaces of subdomains are usually chosen, the number of which is proportional to the number of subdomains. The coarse problem itself can be large and often is a bottleneck of the BDDC computation [9]. Thus an appropriate preconditioning is desired. In paper [9] the authors describe a multilevel BDDC algorithm, where the coarse problem is treated in a similar manner as the original one in the sense that the subdomains are agglomerated into greater sets in the same way as the elements are collected into subdomains. An arbitrary level of agglomeration can be reached by repeating this process. In this paper we suggest a different kind of preconditioning of the coarse problem. We exploit the algebraic multilevel (AML) preconditioning technique. See for example [1] for its description. This is based on a splitting of the space of the base functions into a direct sum of two subspaces. The aim is

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doi:10.1016/j.matcom.2011.06.006

to find such a splitting that the strengthened Cauchy–Buniakowski–Schwarz (CBS) inequality for these two subspaces and for the energy scalar product is valid with a constant less than one. The CBS constant determines the efficiency of the preconditioning. Up to now, the estimates for many kinds of conforming and nonconforming FEs, and for various elliptic operators have been derived [2,3,6,11]. The appropriate AML splittings correspond either to hierarchical bases or to aggregations and differences of FE base functions. In Section 3, we recall the standard AML preconditioning approach. We introduce a theorem which generalizes this strategy for the splitting into more than two subspaces.

In Section 4 the AML preconditioning is applied to the coarse problem in the BDDC algorithm. We exploit two kinds of the coarse DOFs and suggest several kinds of splittings of the coarse spaces into two subspaces. This idea is new. We would like to stress that a variety of coarse DOFs (nodal values, averages over faces or over edges, etc.) can be studied together with appropriate AML splittings. In this paper we introduce only some of them. We provide numerical estimates of the constants in the corresponding strengthened CBS inequality. Moreover, it appears that in some auspicious cases, the coarse space of BDDC can be split into more than two subspaces which are almost pairwise orthogonal with respect to the energy scalar product. This leads to another efficient preconditioning, which is a generalization of the AML principle. Numerical estimates of the parameters of the splitting into either two or more than two subspaces for several two- and three-dimensional problems are presented at the end of Section 4.

2. The BDDC method

We consider a weak formulation of an elliptic problem of the second order defined on a domain Ω with either Dirichlet or Dirichlet and Neumann boundary conditions. The FE discretization reads to find $u \in V$ such that

$$a(u, v) = f(v), \quad v \in V, \tag{1}$$

where V is the space generated by FE base functions. We will denote the functions of V and vectors of their coefficients with respect to the base functions with the same letter.

Let Ω be partitioned into subdomains Ω_i, $i = 1, \ldots, n$. Evaluating the formulae $a(u, v)$ and $f(v)$ over the individual subdomains leads to n separate problems, some of them indefinite. We assign subscripts to the FE base functions on all separated subdomains Ω_i and to corresponding DOFs according to locations of the corresponding nodes in the following manner. Subscript o is connected with internal nodes of the subdomains. Subscript c means coarse nodes. They lie on interfaces of subdomains and define coarse DOFs. Remaining nodes on interfaces of all subdomains have subscript r.

After the appropriate numbering the nodes and after the assembling the blocks according to individual subdomains, we get a matrix of the system of algebraic equations of (1) in the form

$$K = \begin{pmatrix} K_o & K_{or} & K_{oc} \\ K_{or}^T & K_r & K_{rc} \\ K_{oc}^T & K_{rc}^T & K_c \end{pmatrix} \tag{2}$$

and a right hand side b. Submatrix K_o is block diagonal and positive definite. Its dimension equals to the number of all internal nodes. Matrix K_c is positive definite and its dimension equals to the number of coarse DOFs. Matrix K_r is positive definite and block diagonal and its dimension is greater than the number of nodes on interfaces. More precisely, each interface DOF gives rise to several rows and columns of K_r depending on how many subdomains the node belongs to. Of course, the choice of the coarse DOFs must preserve nonsingularity of K_r and thus of K.

Eliminating the lower part in the first block column in K we get the Schur complement matrix

$$S = \begin{pmatrix} S_r & S_{rc} \\ S_{rc}^T & S_c \end{pmatrix} = \begin{pmatrix} K_r & K_{rc} \\ K_{rc}^T & K_c \end{pmatrix} - \begin{pmatrix} K_{or}^T \\ K_{oc}^T \end{pmatrix} K_o^{-1} \left(K_{or} \quad K_{oc} \right).$$

This allows to reduce the size of the problem and to consider only the unknowns associated with the interface nodes.

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