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Mathematics and Computers in Simulation 82 (2012) 1551–1571

MATHEMATICS AND COMPUTERS IN SIMULATION

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Original articles

# Global dynamics of two coupled parametrically excited van der Pol oscillators

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Received 9 March 2009; received in revised form 15 September 2010; accepted 27 February 2012
Available online 11 April 2012

**Abstract**

Using a combination of analytical and numerical methods, the global bifurcations and chaotic dynamics of two non-linearly coupled parametrically excited van der Pol oscillators are investigated in detail. With the aid of the method of multiple scales, the slow flow equations are obtained. Based on the slow flow equations, normal form theory and the techniques of choosing complementary space are applied to find the explicit expressions of the simpler normal form associated with a double zero and a pair of pure imaginary eigenvalues. By the simpler normal form, using the global perturbation method developed by Kovacic and Wiggins, the analysis of global bifurcation and chaotic dynamics of two non-linearly coupled parametrically excited van der Pol oscillators is given. The results indicate that there exists a Silnikov type single-pulse homoclinic orbit for this class of system which implies the chaotic motions can occur. Numerical simulations are also given and verify the analytical predictions.


*Keywords:* Van der Pol oscillators; Normal form; Homoclinic orbit; Global bifurcation; Chaos

## 1. Introduction

It is well known that the dynamics of coupled van der Pol oscillators are interesting because of their importance in science and engineering, and have received considerable attention in the past two decades. But to our knowledge, very little work has been done on the global bifurcations and chaotic dynamics of two non-linearly coupled van der Pol oscillators. In this paper, we study the global bifurcations and chaotic dynamics of two non-linearly coupled parametrically excited van der Pol oscillators.

The system of coupled van der Pol oscillators has been investigated by a number of researchers. The conditions for phase-locked periodic solutions in the case of weakly diffuse linear coupling have been obtained by Rand and Holmes [15]. This work has been extended to the case of strong diffuse linear coupling by Storti and Rand [18]. The regions corresponding to phase locking, phase entrainment and phase drift of the coupled oscillators have been constructed by Chakraborty and Rand [4]. The responses of two-degree-of-freedom systems with cubic non-linearities to a combination of parametric resonance of the sum type and the presence of 1:1 internal resonances have been analyzed by Asmis and Tso [1]. The responses of the cases of two-degree-of-freedom and multi-degree-of-freedom

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doi:10.1016/j.matcom.2012.02.006

systems with cubic non-linearities to a parametrically harmonic excitation have been analyzed by Tezak et al. [19]. The slow flow resulting from the application of the asymptotic perturbation method to the system of two non-linearly coupled van der Pol oscillators under parametric resonance has been investigated by Maccari [11]. The transition boundaries which divide the parameter space into a set of regions, corresponding to different types of solutions of two non-linearly coupled parametrically excited van der Pol oscillators, have been investigated by Bi [2]. The presence of a 3:1 internal resonance in a system of two strongly coupled and detuned van der pol oscillators has been investigated by Rompala [16].

In the last two decades, the global bifurcations and chaotic dynamics of high-dimensional nonlinear systems have received much attention. Wiggins [20] divided four-dimensional perturbed Hamiltonian systems into three types and utilized the Melnikov method to investigate the global bifurcations and chaotic dynamics of these systems. Kovacic and Wiggins [9] developed a new global perturbation technique which may be used to detect the existence of the Silnikov type single-pulse homoclinic and heteroclinic orbits in four-dimensional autonomous ordinary differential equation systems. Using this method, they gave an application to the forced and damped sine-Gordon equation. Combining the higher-dimensional Melnikov method, geometric singular perturbation theory and transversality theory, Haller and Wiggins [8] studied the existence of homoclinic and heteroclinic orbits in a class of near integrable Hamilton systems.

Feng and Sethna [6] used the global perturbation method to study the global bifurcations and chaotic dynamics of a thin plate under parametric excitation and obtained the conditions in which the Silnikov type homoclinic orbits and chaos can occur. Feng and Liew [7] analyzed the existence of Silnikov homoclinic orbits in a perturbed mechanical system by using the global perturbation method. Zhang [21] investigated the global bifurcations and chaotic dynamics for parametrically excited simply supported rectangular thin plates. Zhang and Tang [22] investigated the global bifurcations and chaotic dynamics of the suspended elastic cable under combined parametric and external excitations. Zhang et al. [23] studied the global bifurcations and chaotic dynamics for the nonlinear nonplanar oscillations of a cantilever beam. Recently, the global bifurcations and chaotic dynamics of a string-beam coupled system subjected to parametric and external excitations were investigated by Cao and Zhang [3]. In [25], Zhang et al. used the global perturbation method to study global bifurcations and chaotic dynamics for a rotor-active magnetic bearing system with time-varying stiffness.

The aim of this paper is to discuss the global bifurcations and chaotic dynamics of two non-linearly coupled parametrically excited van der Pol oscillators. The studies are focused on the case of 1:1 internal resonance and primary parametric resonance. Firstly, using the method of multiple scales, the parametrically excited two-degree-of-freedom nonlinear system is transformed to the slow flow equations. Then, from the slow flow equations, the explicit formulas of the simpler normal form associated with a double zero and a pair of pure imaginary eigenvalues are found by normal form theory and the techniques of choosing complementary space. Finally, the global perturbation method presented by Kovacic and Wiggins [9] is employed to analyze the global bifurcations and chaotic dynamics for the two non-linearly coupled parametrically excited van der Pol oscillators. The analysis indicates that there exist Silnikov type single-pulse homoclinic orbit in the slow flow equations. Numerical simulations also show that the chaotic motion can occur in the two non-linearly coupled parametrically excited van der Pol oscillators.

## 2. Slow flow equations and normal form

Nayfeh [12,13] has investigated van der Pol oscillators. Sanchez and Nayfeh [17] have analyzed non-linear parametrically excited one-degree-of-freedom systems. And in [11], Maccari has investigated two non-linearly coupled van der Pol oscillators with only one parametric excitation frequency. Bi [2] has investigated the dynamics of two non-linearly coupled van der Pol oscillators with two different excitation frequencies. The present work is aimed at considering the global bifurcation and chaotic dynamics for two non-linearly coupled van der Pol oscillators with two parametric excitation terms, which can be expressed in the following form [2]:

$$\frac{d^2x}{dt^2} + (\omega_1^2 - 2\epsilon f_1 \cos(\Omega_1 t))x - \epsilon x(x^2 + y^2) + \epsilon(\mu_1 + x^2 + ay^2)\frac{dx}{dt} = 0, \tag{1a}$$

$$\frac{d^2y}{dt^2} + (\omega_2^2 - 2\epsilon f_2 \cos(\Omega_2 t))y - \epsilon y(x^2 + y^2) + \epsilon(\mu_2 + bx^2 + y^2)\frac{dy}{dt} = 0, \tag{1b}$$