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Original article

Predicting future lifetime based on random number of three parameters Weibull distribution

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Abstract

In this paper, two pivotal quantities are modified to construct prediction intervals for future lifetime based on random number of three parameters Weibull distribution, which can be widely applied in reliability theory and lifetime problems. The case of fixed sample size is presented as a special case. The random number has one of three important distributions as special cases. An algorithm is constructed to explain the importance of the theoretical results in applications. Simulation studies are conducted to investigate the efficiency of the purposed results. Finally, two numerical examples for real lifetime data are presented to illustrate the paper.

AMS Classification: Primary 60F05; 62E20; Secondary 62E15; 62G30

Keywords: Order statistics; Truncated distributions; Predicative interval; Probability coverage; Monte Carlo simulation

1. Introduction

There are three main problems in statistical inference, the first is to fit real data to a specific statistical model, the second is to estimate the unknown model parameters and the third is to give methods for predicting future observations based on this model. In many practical applications such as biological science, physics, engineering and manufacture, the available data can be interpreted as lifetimes and it is important to predict future observations. In the theory and methods as well as in various fields of applied statistics, the Weibull distribution has been steadily growing for more than half a century. Moreover, Weibull distribution without any doubt is one of the most important models in modern statistics because of its ability to fit data from various fields, ranging from life data to weather data or observations made in economics and business administration, in hydrology, in biology or in the engineering sciences.

A commonly used model in reliability theory and lifetime studies is the three-parameters Weibull distribution, which was introduced by the Swedish statistician Waloddi Weibull and used for the first time in 1939 in connection with his studies on the strength of materials. For more details and applications of Weibull distribution see Rinne [14].

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A random variable X is said to have three-parameters Weibull distribution, denoted by $W(a, b, c)$, if its probability density function (pdf) is given by,

$$f(x)=\begin{cases}\dfrac{c}{b}\left(\dfrac{x-a}{b}\right)^{c-1}\exp\left[-\left(\dfrac{x-a}{b}\right)^{c}\right], & x>a,\\ 0, & x\le a,\end{cases} \tag{1}$$

where $a\in\mathbb{R}$ is the location parameter, $b>0$ is the scale parameter and $c>0$ is the shape parameter. The corresponding distribution function (df) is given by

$$F(x)=1-\exp\left[-\left(\frac{x-a}{b}\right)^{c}\right],\quad x\ge a. \tag{2}$$

The order statistics play an important role for the lifetime prediction methods because if m items such as radio tubes, wire fuses, or light bulbs are put simultaneously in a life test, the weakest component will fail first, followed by the second weakest, and so on until all have failed. In biological sciences, we are interested in the time to death after n animals are subjected to a common dose of radiation. In such cases, the observations arrive in ascending order of magnitude and do not have to be ordered after collection of the data. The practical importance of such experiments is evident. Moreover, the possibility is now open of terminating the experiment before its conclusion, by stopping after a given time (Type I censoring) or after a given number of failures (Type II censoring). It may be of interest, to predict the time at which all the components will have failed or to predict the mean failure time of the unobserved lifetimes. In these cases the interval or point prediction may be described, for more details of the theory and applications of order statistics see David and Nagaraja [3].

The estimation and prediction problems have been studied by many authors, among of them are Lawless ([6,7]), Lingappaiah ([9–11]), Geisser [5], Yildirim [18], Yuen and Tse [19] and Yang et al. [17]. Recently, Wang and Wang [16] studied the prediction problem of the life-span of a system whose components connected in series and the lifetime of the components follows the exponential distribution. Aboeleneen [1] considered Bayesian and non-Bayesian methods for estimating Weibull parameters based on generalized order statistics.

In this paper, I develop two pivotal statistics, which can be considered as basic tools for constructing the prediction intervals, namely,

$$U_{r,s}=\frac{(X_{s:n}-a)^{c}-(X_{r:n}-a)^{c}}{(X_{r:n}-a)^{c}},\quad 1\le r<s\le n \tag{3}$$

and

$$V_{r,s}=\frac{(X_{s:n}-a)^{c}-(X_{r:n}-a)^{c}}{b^{c}\,T_{r:n}},\quad 1\le r<s\le n, \tag{4}$$

where $X_{1:n}<X_{2:n}<\cdots<X_{n:n}$ is the order statistics of the random sample $X_1, X_2, \cdots, X_n$; X_j, $j=1, 2, \ldots, n$, follow $W(a, b, c)$ and $T_{r:n}=\sum_{j=1}^{r}Z_j,\ Z_j,\ j=1,2,\ldots,r$ are given by

$$Z_i=(n-i+1)\left[\left(\frac{X_{i:n}-a}{b}\right)^{c}-\left(\frac{X_{i-1:n}-a}{b}\right)^{c}\right],\quad i=1,2,\ldots,n\quad\text{with}\quad X_{0:n}\equiv a. \tag{5}$$

The above pivotal quantities are applied to construct prediction intervals for future observations from three parameters Weibull distribution, when the sample size is assumed to be positive integer-valued random variable independent of the observations.

The rest of the paper is organized as follows. In Section 2 the theoretical results are presented and Section 3 contains simulation studies for some special cases of the random sample size. Two numerical examples for real lifetime data are presented in Section 4.

2. The main results

In this section, the df's of the two statistics, $U_{r,s}$ and $V_{r,s}$ are derived in Theorems 1 and 2 respectively.

The following lemma express an interesting fact that can be applied for solving many problems, and it will be used in the proof of Theorem 2.