



## Original articles

# A bifurcation path to chaos in a time-delay fisheries predator–prey model with prey consumption by immature and mature predators

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## Abstract

Stage-structure models have been extensively applied in predator–prey systems. In this paper, we consider an application to fisheries. We assume that there is a single prey population and a predator population that can be separated by reproduction ability into an immature and a mature stage, with a time delay for the immature to mature transition. Our model includes the new assumption that both predators are able to hunt the same prey, although at different rates. It is proved that the system has three nonnegative equilibrium points, namely, a trivial point with all populations zero, a predator-free equilibrium point, and a coexistence equilibrium point with all three populations non-zero. It is proved that the trivial equilibrium point is always unstable, that the predator-free equilibrium point is stable if and only if the coexistence equilibrium point does not exist, and that the coexistence point can either be stable for all time delays or become unstable if a Hopf bifurcation exists at a critical time delay. Numerical simulations show that the behavior of the system can become extremely complicated as the time delay is increased, with the long-time behavior changing from a stable coexistence equilibrium, to a limit cycle with one local maximum and minimum per cycle (Hopf bifurcation), to limit cycles with an increasing number of local maxima and minima per cycle, and finally to chaotic-type solutions.

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## 1. Introduction

Many studies (see, e.g. [7–9,18,21,32,33,39,46]) have shown that the age structure and stage structure of a fish population are important factors in determining the growth rate of a fish population and the sustainability of a fishery. Stage-structured models have been of much interest in recent years (see, e.g. [24]). These models have been applied to both single species and multispecies populations. In 1990, Aiello and Freedman [1] developed a stage-structured

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single-species population model with time delay. They divided the population into two stages (immature and mature populations) and used a time delay for conversion from immature to mature stage. Song and Chen [36] modified this model by including three different harvesting policies of the mature predator, i.e., constant, variable, and periodic. For a multispecies model, the stage-structure has been applied in a prey population [2,11,26], in a predator population [12,13,20,19,29,34,38,40–43,45,47], or in both populations [17,24,25,35]. Gourley and Kuang [16] have made a detailed study of the dependence of the dynamical behavior of a time-delay multispecies model as a function of maturation delay and death rate.

For a predator–prey model with stage-structure for predator, many researchers have made the important assumption that only mature predators have reproductive ability and can attack prey. The given reason is that, in some species such as mammals, the immature are cared for by their parents and have no need to find their own food. Wang and coworkers [40,41,43,44] have developed a model under these assumptions and used a Holling type II response function for the consumption rate of the mature.

The Wang model has been proved to have properties of boundedness, permanence, existence of periodic solutions, and global stability of a positive equilibrium point. Many authors (see, e.g., [14,13,45]) have studied the Wang model and given proofs of important properties of the model. The model has also been modified in various ways. For example, Wang et al. [43] used Holling type III as the response function, and Shi et al. [34] studied the model with the Crowley–Martin response function.

In previous papers [4,5], we generalized the stage-structured model of Wang [40] by assuming that both the immature and mature stages of the predator are able to hunt the same prey population. The reason is that most commercial marine fish abandon their fertilized eggs after spawning. Therefore, newborn fish have to seek their food by themselves. In addition, some fish mature slowly such as Pacific Bluefin tuna which spend about 2–4 years to reach maturity [37]. Without predation, it is not possible for the immature stage to survive. There are several papers in the literature (see, e.g., [6,15,31]) that give examples of immature and mature predators that consume the same prey species, but at different rates. For example, Gerasimova [15] has given a detailed study of the spring feeding of Capelin fish on the Grand Bank in which she found that both immature and mature Capelin eat copepods, but in different amounts. She also found that mature Capelin eat other species such as euphausiid and can also eat immature Capelin. In this paper, we will consider the special case in which the mature stage and immature stage eat the same prey as the behavior of this special case is already very complicated.

Motivated by the work of Feng and Hongwei [11], we are concerned with the effect of time delay due to the transformation of the immature stage into the mature stage. In this paper, we extend the model of the previous papers [4,5] to include a time delay for conversion from immature to mature stage and then examine the effects of changing the time delay from a short delay (fish mature fast) to a long delay (fish mature slowly). An understanding of the effects of different time delays can help us to explain population dynamics under different situations and monitor the sensitivity and recovery of predator populations.

It has also been shown by several authors that predator–prey models can show very complicated dynamical behavior of the type that we have found for our model. For example, Zeng et al. [48] have shown the bifurcations that occur in a delayed predator–prey model with a single predator stage, impulsive harvesting and Holling type II functional response.

The organization of this paper is as follows. In Section 2, we describe the stage structured time delay model and find three equilibrium points, namely, trivial, predator-free and coexistence. In Section 3, we analyze the stability of the trivial and predator-free equilibrium points. In Section 4, we analyze the stability of the coexistence equilibrium point and find conditions for the existence of Hopf bifurcations. In Section 5, we discuss the results of numerical simulations which show that the system has a “bifurcation path to chaos”. Finally, Section 6 contains a discussion and conclusions.

## 2. The stage-structured time-delay model and equilibrium points

In this section, we describe and analyze a simple stage-structured model for a prey population  $x(t)$ , an immature predator population  $y_1(t)$  and a mature predator population  $y_2(t)$ , where  $t$  is time. We assume that both immature and mature predators eat the same prey populations and that there is a time delay  $\tau > 0$  for the immature predator to reach maturity. This model is mathematically interesting as it has very complicated dynamical behavior. We consider the

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