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Mathematics and Computers in Simulation 82 (2012) 1531–1543

MATHEMATICS AND COMPUTERS IN SIMULATION

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Original Articles

Mean square exponential stability of impulsive stochastic reaction-diffusion Cohen–Grossberg neural networks with delays

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Received 27 December 2009; received in revised form 9 September 2010; accepted 4 November 2011
Available online 4 April 2012

Abstract

In this paper, we establish a method to study the mean square exponential stability of the zero solution of impulsive stochastic reaction-diffusion Cohen–Grossberg neural networks with delays. By using the properties of M-cone and inequality technique, we obtain some sufficient conditions ensuring mean square exponential stability of the zero solution of impulsive stochastic reaction-diffusion Cohen–Grossberg neural networks with delays. The sufficient conditions are easily checked in practice by simple algebra methods and have a wider adaptive range. Two examples are also discussed to illustrate the efficiency of the obtained results.

Keywords: Mean square exponential stability; Impulsive; Stochastic reaction-diffusion system; Delays

1. Introduction

Since Cohen–Grossberg neural network was first proposed by Cohen and Grossberg [7] in 1983, many researchers have done extensive works on this subject due to their extensive applications in many fields such as pattern recognition, parallel computing, associative memory, signal and image processing and combinatorial optimization. In such applications, it is of prime importance to ensure that the designed neural networks is stable. The stability analysis problem for Cohen–Grossberg neural networks has gained much research attention, and a large amount of results related to this problem have been published, (see, e.g., [5,14,17,19,30,32,24,20,27]).

However, strictly speaking, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields. Therefore we must consider that the activations vary in space as well as in time. In [15,16,25,33,13], the authors have considered the stability of neural networks with reaction-diffusion terms.

On the other hand, a real system is usually affected by external perturbations which in many cases are of great uncertainty and hence may be treated as random, as pointed out by Haykin [12] that in real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. It has also been known that a neural network could be stabilized or destabilized by certain

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0378-4754/$36.00
http://dx.doi.org/10.1016/j.matcom.2011.11.007

stochastic inputs. Hence, the stability analysis problem for stochastic neural network becomes increasingly significant, and some results on stability have been derived, (see, e.g., [21,22,28,29,31]).

However, taking more factors into account leads to the development of the theory of impulsive partial differential equations (PDE). This theory marked its beginning with the paper [8]. The impulsive PDE provides natural framework for mathematical simulation of numerous processes and phenomena in theoretical physics, population dynamics, biotechnologies, chemistry, impulse technique and economics. For the present, the theory of the impulsive PDE has been an object of many researchers, we refer the readers to the monograph [1] and the papers [2,3,10,9,11].

Very recently, Luo and Zhang [18] have employed the Lyapunov direct method to investigate the stability of Itô stochastic reaction diffusion systems. Wan and Zhou [23] studied the mean value exponential stability of an equilibrium solution of stochastic reaction-diffusion Cohen–Grossberg neural networks with delays by using some inequality technique.

To the best of our knowledge, however, few authors have considered the exponential stability for impulsive stochastic Cohen–Grossberg neural networks with delays and diffusion terms. Motivated by the above discussions, our objective in this paper is to present the sufficient conditions ensuring the mean square exponential stability of impulsive stochastic Cohen–Grossberg neural networks with delays and diffusion terms.

2. Model description and preliminaries

For convenience, we introduce several notations and recall some basic definitions.

Let R^n be the space of n-dimensional real column vectors, $\mathcal{N} \triangleq \{1, 2, \cdots, n\}$, $R_+ \triangleq [0, +\infty)$, and $R^{m\times n}$ denotes the set of $m \times n$ real matrices. Usually I denotes an $n \times n$ unit matrix. For $A, B \in R^{m\times n}$ or $A, B \in R^n$, the notation $A \geq B$ $(A > B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$\geq (>)$". Especially, $A \in R^{m\times n}$ is called a nonnegative matrix if $A \geq 0$, and $z \in R^n$ is called a positive vector if $z > 0$.

$C[X, Y]$ denotes the space of continuous mappings from the topological space X to the topological space Y. Especially, let $C \triangleq C\,[[-\tau, 0]\,, R^n]$.

$$PC[J, H] = \{\psi(t) : J \to H \mid \psi(t) \text{ is continuous for all but at most countable points } s \in J \text{ and at these points } s \in J,\ \psi(s^+) \text{ and } \psi(s^-) \text{ exist, } \psi(s^+) = \psi(s)\},$$

where $J \subset R$ is an interval, H is a complete metric space, $\psi(s^+)$ and $\psi(s^-)$ denote the right-hand and left-hand limit of the function $\psi(s)$, respectively. Especially, let $PC \triangleq PC\,[[-\tau, 0]\,, R^n]$.

For $\varphi \in PC$ or $\varphi \in C$, we define

$$[\varphi(t)]_\tau = \left([\varphi_1(t)]_\tau, \cdots, [\varphi_n(t)]_\tau\right)^T, \qquad [\varphi(t)]_\tau^+ = [|\varphi(t)|]_\tau, \qquad [\varphi_i(t)]_\tau = \sup_{-\tau\leq\theta\leq 0} \varphi_i(t+\theta), \quad i \in \mathcal{N},$$

and $D^+\varphi(t)$ denotes the upper right derivative of $\varphi(t)$ at time t.

Consider the following impulsive stochastic Cohen–Grossberg neural networks with delays and diffusion terms

$$\begin{cases} dy_i(t,x) = \displaystyle\sum_{k=1}^{m} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k}\right)dt - d_i(y_i(t,x))\left(c_i(y_i(t,x)) - \sum_{j=1}^{n} a_{ij} f_j\left(y_j(t,x)\right)\right. \\ \quad \left. - \displaystyle\sum_{j=1}^{n} b_{ij} g_j\left(y_j\left(t-\tau_{ij}(t), x\right)\right)\right)dt + \displaystyle\sum_{j=1}^{n} \sigma_{ij}\left(y(t,x), y\left(t-\tau_{ij}(t), x\right)\right) dw_j(t),\ (t,x) \in \Gamma, \\ \dfrac{\partial y_i}{\partial n} := \left(\dfrac{\partial y_i}{\partial x_1}, \cdots, \dfrac{\partial y_i}{\partial x_m}\right)^T = 0, \quad (t,x) \in H, \\ y_i(t_0+s, x) = \varphi_i(s,x), -\tau \leq s \leq 0, \quad x \in X, \\ y_i(t_k, x) = h_{ik}\left(y_1\left(t_k^-, x\right), \cdots, y_n\left(t_k^-, x\right)\right), \quad x \in X, \quad k = 1, 2, 3, \cdots, \end{cases} \tag{1}$$

for $i \in \mathcal{N}$. In the above model, $n \geq 2$ is the number of neurons in the network; x_i is space variable; $y_i(t, x)$ is the state variable of the ith at time t and in space x; $f_j(y_j(t, x))$ and $g_j(y_j(t, x))$ denote the activation functions of the jth unit at time t and in