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Controlling chaos in a food chain model

Anuraj Singh[a,*], Sunita Gakkhar[b]

[a] *Department of Physics and Astrophysics, University of Delhi, Delhi, India*
[b] *Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee 247667, India*

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Abstract

In this paper, different mechanisms are used to suppress the chaos in a food chain model. The control is applied to the chaotic system so as the controlled system admits a stable attractor which may be an equilibrium point or a limit cycle. The bounded feedback is used to achieve the stabilization of unstable fixed point of the uncontrolled chaotic system. Delayed feedback control is used to control the chaos to periodic orbits. Numerical results substantiate the analytical findings.

Keywords: Food chain model; Chaos; Unstable periodic solution; Feedback control

1. Introduction

Controlling chaos is a focal issue in the nonlinear problems ranging from physics and chemistry to biology and economics. Chaos control concept has come to mean the stabilization of unstable periodic orbits (including unstable equilibria) of dynamical systems. In a broader sense, controlling chaos can be understood as a process or mechanism which enhances existing chaos or creates chaos in a dynamical system when it is useful or beneficial. The control of chaos involves eliminating and weakening of chaos when it is undesirable and harmful. In such cases, it is usually a process of stabilizing unstable periodic orbits or reducing the leading positive Lyapunov exponent of the dynamical chaotic system, [6].

The food chains and food webs consist of three or more species have all the ingredients for the occurrence of chaos. This has motivated several investigators to look for chaotic dynamics in ecosystem, [9,14,26,32]. However, the ecological systems are robust which can withstand the perturbations of the natural habitats. In contrast, the chaotic system is unpredictable and sensitive to initial conditions. Chaos is frequently discussed in ecological models but rarely observed in natural population. It is also observed that the chaotically fluctuating population is prone to extinction, with consequence that group selection acts to eliminate species and chaos disappears [23]. It is desirable to exert suitable control to stabilize the unstable attractor. In physical and engineering systems, a lot of work has been carried out in chaos control. With reference to ecological system, very little has been done so far [4,27].

* Corresponding author.
E-mail addresses: anurajiitr@gmail.com (A. Singh), sungkfma@iitr.ernet.in (S. Gakkhar).

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Naji and Balasim investigated three species food chain model with Beddington–DeAngelis functional response [18]. They exhibited chaotic dynamics for realistic and biologically feasible parameters values. They also observed that adding small amount of constant immigration to prey species stabilize the system. In another investigation, three species food chain model with Crowley–Martin type functional response is taken [29]. The results showed that the system exhibited rich complexity features such as stable, periodic and chaotic dynamics, from the theoretical and numerical responses. Three-species food chain model with Beddington–DeAngelis nonlinear functional response has been investigated as well [33]. This paper demonstrates the presence of chaos through strange attractor and computation of the largest Lyapunov exponent. Multi-species food chain model have also been studied extensively and literature [9,11,13,17,20] has shown such models exhibit chaotic behavior under a Holling-type or Beddington–DeAngelis non-linear functional response [1,3,12,30,31].

The control strategy may be manmade or influenced by nature/surroundings. The inclusion of additional predator may control the chaos in Hastings Powell food chain, [10]. Control mechanism can be applied externally as in case of harvesting or stocking of species, [15,16]. For chaos control, two categories of control mechanisms are used: feedback and non-feedback. The feedback methods, (see [5,21,25]) are primarily devised to control chaos by stabilizing a desired unstable periodic orbit embedded in a chaotic attractor. The non-feedback methods, (see [19,24]) suppress chaotic orbits by converting the system dynamics to a periodic orbit. A bounded feedback method (see [7]) preserves the steady states of the original system. It vanishes after stabilization is achieved.

The delayed feedback control method is given by, Pyragas (see [22]) to control chaos in continuous dynamical system. An advantage of this approach is that it does not require any other control force input, nor access to system parameters. Also, it may not need to exactly calculate the target trajectory. The DFC method allows one to stabilize unstable periodic orbits of a strange attracted over a large range of parameters. In this paper, another method of control based on DFC method: approximated delayed feed back method is proposed. The controlled system is converged to a small periodic orbit.

2. System of coupled dynamics

Let $X(t')$ be the population density of prey at the lowest level of a tri trophic food chain at time t'. Let $Y(t')$ and $Z(t')$ be the population densities of intermediate and top predators at middle and highest trophic levels respectively. The dynamics of three species food chain model with Beddington–DeAngelis type of functional response is governed by the system of differential equations, Naji and Balasim [18]:

$$\begin{aligned} \frac{dX}{dt'} &= RX\left(1-\frac{X}{K}\right) - F_1(X,Y)Y \\ \frac{dY}{dt'} &= E_1F_1(X,Y)Y - F_2(Y,Z)Z - D_1Y \\ \frac{dZ}{dt'} &= E_2F_2(Y,Z)Z - D_2Z, \qquad F_i(U,V) = \frac{A_iU}{B_iV+U+C_i}, \quad i=1,2. \end{aligned} \tag{1}$$

The model parameters R, K, A_i, B_i, C_i, D_i and E_i, $i = 1, 2$ assume only positive values and are defined as follows:

R is the intrinsic growth rate and K is the carrying capacity of the prey species. The constants D_i $(i = 1, 2)$ describe the loss of predator population in absence of food. The predator Y consumes the prey X with functional response $F_1(X, Y)$, while predator Z preys upon Y according to functional response $F_2(Y, Z)$. The model considers the Beddington–DeAngelis type of functional response. The constants A_i, B_i and C_i are functional response parameters. The constants E_1, E_2 are conversion rates of prey into predator for species Y and Z respectively.

The model (1) has 12 parameters which are reduced to 8 by introducing the following non-dimensional variables and parameters:

$$x=\frac{X}{K},\quad y=\frac{A_1Y}{RK},\quad z=\frac{A_1A_2Z}{R^2K},\quad t=Rt'$$
$$w_1=\frac{B_1R}{A_1},\quad w_2=\frac{C_1}{K},\quad w_3=\frac{E_1A_1}{R},\quad w_4=\frac{B_2R}{A_2},\quad w_6=\frac{D_1}{R},\quad w_7=\frac{E_2A_2}{R},\quad w_8=\frac{D_2}{R}.$$