Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 79 (2009) 2361–2372
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

The efficient solution of direct medium problems by using translation techniques

N. Egidi *, P. Maponi

Università di Camerino, Dipartimento di Matematica e Informatica, Camerino(MC) 62032, Italy

Available online 31 January 2009

Abstract

We consider a Fredholm integral equation arising from a time-harmonic electromagnetic scattering problem for inhomogeneous media. The discretization of this equation usually produces a large dense linear system that must be solved by iterative methods. To speed up these methods we propose an efficient computation of the action of the corresponding coefficient matrix on a generic vector. This computation is mainly based on the well known addition formula for the Hankel functions and a simple translation argument. We present some numerical examples to show the efficiency of the proposed method.

Keywords: Electromagnetic scattering; Fredholm integral equation; Iterative method

1. Introduction

We consider a scattering problem for electromagnetic waves in inhomogeneous media. In particular, we suppose that the scattered electromagnetic wave is generated from the interaction of an incident electromagnetic wave and an inhomogeneity in the corresponding propagation medium. From the knowledge of the incident field and of the inhomogeneity, the computation of the scattered field can be performed by solving a boundary value problem for the Maxwell equations, see [2] page 238 for details. However, suitable assumptions on the electromagnetic fields and on the inhomogeneity in the medium allow to obtain a simplified formulation of this boundary value problem. More precisely, the incident wave and the scattered wave are assumed to be time-harmonic with the same time-frequency; moreover, for the refractive index of the inhomogeneity and for the incident wave the usual Transverse Magnetic (TM) symmetry settings are taken into account, see [10] chap. 6, and [3] for details. From these assumptions we have that the space-variables dependent part of the electromagnetic fields can be described by scalar functions of two variables, see [3], [7] for details.

We introduce the notation. Let $\mathbb{R}$, $\mathbb{C}$ be the sets of real numbers, and complex numbers, respectively. Let N be a positive integer, let $\mathbb{R}^N$, $\mathbb{C}^N$ be the N-dimensional real Euclidean space, and the N-dimensional complex Euclidean space, respectively. Let A be a finite set, we denote with $\#(A)$ the number of elements in A. Let $\underline{x}, \underline{y} \in \mathbb{R}^N$, we denote with $\underline{x}^t \underline{y}$ the Euclidean scalar product of z $\underline{x}$ and $\underline{y}$, the superscript t means transposed, $||\underline{x}||$ denotes the Euclidean norm of $\underline{x}$. Let M be a positive integer, we denote with $\mathcal{M}_{\mathbb{C}}(M, N)$ the space of complex matrices having M rows and N columns. Let $\mathbb{S} = \{\underline{x} \in \mathbb{R}^2 : ||\underline{x}|| = 1\}$. Let $z \in \mathbb{C}$, we denote with $\mathrm{Re}(z)$,$\mathrm{Im}(z)$ the real part and the imaginary part of z, respectively. Finally, we denote with ι the imaginary unit.

* Corresponding author.
E-mail addresses: nadaniela.egidi@unicam.it (N. Egidi), pierluigi.maponi@unicam.it (P. Maponi).

0378-4754/$36.00
doi:10.1016/j.matcom.2009.01.010

Let

$$u^i(\underline{x}) = e^{\imath k \underline{\alpha}^t \underline{x}}, \ \underline{x} \in \mathbb{R}^2, \tag{1}$$

be the scalar function describing the incident field; note that this is a plane wave with propagation direction $\underline{\alpha} = (\alpha_1, \alpha_2)^t \in \mathbb{S}$, and wave number $k > 0$. Let $n(\underline{x}) \in \mathbb{C}$, $\underline{x} \in \mathbb{R}^2$, be the refractive index of the medium. Let $m(\underline{x}) = 1 - n(\underline{x})$, $\underline{x} \in \mathbb{R}^2$. We suppose that there exists a compact domain $D \subset \mathbb{R}^2$, such that $n(\underline{x}) = 1$, $\underline{x} \in \mathbb{R}^2 \setminus D$. From the integral formulation of the scattering problem we have that $u^s : \mathbb{R}^2 \to \mathbb{C}$, the scalar function describing the scattered field, is the solution of the following integral equation:

$$u(\underline{x}) + \frac{\imath k^2}{4} \int_D H_0^{(1)}(k||\underline{x} - \underline{y}||) u(\underline{y}) m(\underline{y})\, d\underline{y} = u^i(\underline{x}), \ \underline{x} \in \mathbb{R}^2, \tag{2}$$

where $u = u^i + u^s$, and $H_0^{(1)}$ is the Hankel function of first kind and order 0, see [1] page 358 for details.

We consider the following problem: given k, $\underline{\alpha}$, n, compute the solution u^s of problem (2).

Formula (2), restricted to $\underline{x} \in D$, gives a Fredholm integral equation of second kind for the unknown function $u(\underline{x})$, $\underline{x} \in D$. Once this equation is solved, from the knowledge of $u(\underline{x})$, $\underline{x} \in D$, we can immediately compute $u(\underline{x})$ for each $\underline{x} \in \mathbb{R}^2$ by using again formula (2). Thus, the numerical solution of the scattering problem under consideration can be obtained by a discretization of integral equation (2) restricted to $\underline{x} \in D$.

We consider a simple discretization of such an integral equation. Let $B \subset \mathbb{R}^2$ be a square containing D. Let ν_1, ν_2 be positive integers, let $\mathcal{J} = \{(i_1, i_2), i_1 = 1, 2, \ldots, \nu_1, i_2 = 1, 2, \ldots, \nu_2\}$, let $\mathcal{B} = \{B_{i_1,i_2} \subset \mathbb{R}^2, (i_1, i_2) \in \mathcal{J}\}$ be a partition of B. We consider the following discretization scheme:

$$u_{i_1,i_2} + \frac{\imath k^2}{4} \sum_{(j_1,j_2) \in \mathcal{J}} u_{j_1,j_2} m_{j_1,j_2} \int_{B_{j_1,j_2}} H_0^{(1)}(k||\underline{\xi}_{i_1,i_2} - \underline{y}||) \mathrm{d}\underline{y} = u^i_{i_1,i_2}, \ (i_1, i_2) \in \mathcal{J}, \tag{3}$$

where, for $(i_1, i_2) \in \mathcal{J}$, $\underline{\xi}_{i_1,i_2} \in \mathbb{R}^2$ is the center of B_{i_1,i_2}, $u^i_{i_1,i_2} = u^i(\underline{\xi}_{i_1,i_2})$, $m_{i_1,i_2} = m(\underline{\xi}_{i_1,i_2})$, and u_{i_1,i_2} is the corresponding approximation of $u(\underline{\xi}_{i_1,i_2})$. Note that these equations arise from a simple piecewise constant approximation of functions u and m with respect to the partition $\mathcal{B}$ of B. This discretization scheme is chosen for the sake of simplicity, however the arguments in the following sections can be provided also for other better quality schemes. Furthermore, we note that the solution u_{i_1,i_2}, $(i_1, i_2) \in \mathcal{J}$ of linear system (3), in accordance with the definition of the total field u (see below formula (2)), is given by two parts: the discretization of the incident field, the discretization of the scattered field. The first part, i.e. $u^i_{i_1,i_2}$, $(i_1, i_2) \in \mathcal{J}$, can be easily computed from (1), the second part is the actual unknown of linear system (3).

Linear system (3) has a dense coefficient matrix, however, due to its dimension, iterative methods are usually considered for the computation of its solution. The efficiency of these methods can be improved in a number of ways, such as for example improving the efficiency of the iterative technique, providing an efficient preconditioner [4,5], decreasing the computational cost of each iterate and so on. We consider a translation technique to decrease the computational cost of each iterate. This technique has been presented in [3], and it is based on the well known addition formula for the Hankel functions. The excellent performances shown by this technique give good reasons for a detailed study of this technique. The efficiency of the translation technique is shown by using some numerical experiments, where integral Eq. (2) is solved for different refractive indices n and for different wave numbers k, by using linear system (3). We note that analogous methods have been already proposed for the solution of other similar problems, see [6], [9] for details.

In Section 2 we present the translation technique, that can be seen as a suitable employment of the addition formula for the Hankel functions in the iterative solution of integral equation (2). In Section 3 we show the results of some numerical experiments. In Section 4 we give some conclusions and future developments of this paper.

2. The translation technique

Linear system (3) is rewritten in the following compact form:

$$(I + HM)\underline{u} = \underline{b}, \tag{4}$$