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Local refinement based on the 7-triangle longest-edge partition

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Abstract

The triangle longest-edge bisection constitutes an efficient scheme for refining a mesh by reducing the obtuse triangles, since the largest interior angles are subdivided. In this paper we specifically introduce a new local refinement for triangulations based on the longest-edge trisection, the 7-triangle longest-edge (7T-LE) local refinement algorithm. Each triangle to be refined is subdivided in seven sub-triangles by determining its longest edge. The conformity of the new mesh is assured by an automatic point insertion criterion using the oriented 1-skeleton graph of the triangulation and three partial division patterns.

Keywords: Local refinement; Longest-edge based algorithms; Skeleton

1. Introduction

In the last years many different strategies have been developed to locally refine triangular and tetrahedral meshes. There are two main steps in local adaptive refinement: the refinement of a subset of elements based on local error indicators, and the consistent transition between refined and unrefined cells [1,3]. We refer to the latter as 'mesh conformity'. Several refinement and improvement techniques for two- and three-dimensional triangulations are available [2,5,13,4,7]. Some of them are based on edge bisection of the elements, triangles or tetrahedra.

In two dimensions, the 4-Triangle Longest-Edge (4T-LE) partition and the associated local refinement was introduced by Rivara in [13,16,14]. This partition was extended to tetrahedral meshes in [8], where the tetrahedral mesh is locally refined after the skeleton of the triangulation, that is, the set of triangular faces of the tetrahedra, has been divided by the 4T-LE partition. The 4T-LE partition of a given triangle t never produces an angle smaller than half the minimum original angle [18], and besides, it shows a remarkable mesh quality improvement [16] between certain limits, as recently studied in [12]. In order to devise a new strategy to improve mesh quality through iterative refinement, in this paper we use a new triangle partition, the seven-triangle longest-edge (7T-LE) partition [6]. This partition, first, positions two equally spaced points per edge and, then, the interior of the triangle is divided into seven sub-triangles in a manner compatible with the subdivision of the edges. Three of the new sub-triangles are similar to the original, two are similar to the new triangle also generated by the 4T-LE, and the other two triangles are, in general, better shaped.

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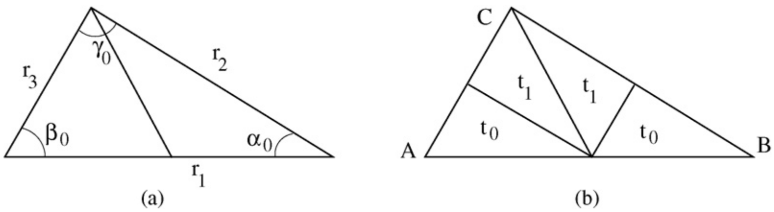

Fig. 1. (a) LE partition of triangle t_0 and (b) 4T-LE partition of triangle t_0.

The outline of this paper is as follows: Section 2 summarizes the 4T-LE partition and its improvement property, showing the limitations of this improvement. Section 3 presents the new 7T-LE partition and shows in a geometric diagram that the new triangles generated by the 7T-LE partition are better shaped that those generated by the 4T-LE partition. Section 4 introduces the new local refinement algorithm based on the 7T-LE partition, by using the 1-skeleton oriented graph approach [20]. Also the inverse counterpart algorithm for derefining a sequence of triangulation is presented. The paper closes with some conclusions and ongoing research.

2. The 4T-LE partition and its improvement property

Here the 4-triangle longest-edge partition and its self-improvement is summarized. The section closes with an example illustrating the limits of the characteristic improvement.

In the following, for any triangle t, the edges and angles will be respectively denoted in decreasing order $r_1 \geq r_2 \geq r_3$, and $\gamma \geq \beta \geq \alpha$. Furthermore, $t(\alpha, \beta, \gamma)$ will be the class of similar triangles with angles $\gamma \geq \beta \geq \alpha$. Interchangeably, t will represent an element of the class $t \in t(\alpha, \beta, \gamma)$ or the class itself.

Definition 1. (edge bisection and longest-edge bisection) The longest-edge (LE) partition of a triangle t_0 is obtained by joining the midpoint of the longest edge of t_0 with the opposite vertex (Fig. 1 (a)). The 4-triangle longest-edge (4T-LE) partition is obtained by joining the midpoint of the longest edge to the opposite vertex and to the midpoints of the two remaining edges (see Fig. 1(b)).

Note that the two subtriangles with edges coincident with the longest edge of the parent are similar to the parent. The remaining two subtriangles form a similar pair that, in general, are not similar to the parent triangle. We refer to such new triangle shapes as 'dissimilar' to those preceding.

Since the first 4T-LE partition of any triangle t_0 introduces two new edges parallel to the edges of t_0, the first 4T-LE partition of a single triangle t_0 produces two triangles similar to t_0 and two (potentially) new similar triangles t_1; and, consequently, the iterative 4T-LE partition of any triangle t_0 introduces (at most) one new dissimilar triangle per iteration [16].

The iterative 4-triangle longest-edge partition produces a finite sequence of 'better' triangles satisfying the properties illustrated in the following diagram until triangle t_N becomes non-obtuse, where α_I, and γ_I are respectively the smallest and the largest angles of triangle t_i. The arrow emanating from triangle t_i to triangle t_{i+1} means that the (first) 4-triangle longest-edge partition of triangle t_i produces the new dissimilar triangle t_{i+1}:

$$
\begin{array}{ccccc}
t_0 & \rightarrow & t_1 & \rightarrow \dots \rightarrow & t_N \\
\text{(obtuse)} & & \text{(obtuse)} & & \text{(non-obtuse)} \\
\alpha_0 & & \alpha_1 > \alpha_0 & & \alpha_N > \alpha_{N-1} \\
\gamma_0 & & \gamma_1 \leq \gamma_0 - \alpha_1 & & \gamma_N \leq \gamma_{N-1} - \alpha_N
\end{array}
$$

Diagram 1 (from [16])

We refer the reader to [12] in which the self-improvement property of the 4T-LE partition has been studied in detail. By way of example, consider the initial triangle t_0 with angles $(\alpha, \beta, \gamma) = (1.95, 32.595, 145.455)$. The evolution of the smallest angle and largest angle for each of the new triangles generated by the 4T-LE partition is shown in Fig. 2.