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Bayesian analysis of stochastic volatility models with mixture-of-normal distributions

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Abstract

Stochastic volatility (SV) models usually assume that the distribution of asset returns conditional on the latent volatility is normal. This article analyzes SV models with a mixture-of-normal distributions in order to compare with other heavy-tailed distributions such as the Student-t distribution and generalized error distribution (GED). A Bayesian method via Markov-chain Monte Carlo (MCMC) techniques is used to estimate parameters and Bayes factors are calculated to compare the fit of distributions. The method is illustrated by analyzing daily data from the Yen/Dollar exchange rate and the Tokyo stock price index (TOPIX). According to Bayes factors, we find that while the t distribution fits the TOPIX better than the normal, the GED and the normal mixture, the mixture-of-normal distributions give a better fit to the Yen/Dollar exchange rate than other models. The effects of the specification of error distributions on the Bayesian confidence intervals of future returns are also examined. Comparison of SV with GARCH models shows that there are cases that the SV model with the normal distribution is less effective to capture leptokurtosis than the GARCH with heavy-tailed distributions.

Keywords: Bayes factor; Mixture-of-normal distributions; Generalized error distribution; Marginal likelihood; Markov-chain Monte Carlo; Multi-move sampler; Student-t distribution

1. Introduction

It has long been recognized that daily asset returns are leptokurtic, and hence some authors model stock returns as i.i.d. draws from fat-tailed distributions; see Mandelbrot [27] and Fama [18]. It is also a well-known phenomenon that the asset return volatility changes randomly over time. If so, the unconditional distribution is leptokurtic even though the conditional distribution is normal; see Bollerslev et al. ([8], p. 2963). Note, however, that it does not mean that the leptokurtosis of asset returns can fully be explained by changes in volatility. Actually, several authors have found that the conditional distribution is also leptokurtic by assuming leptokurtic distributions for the conditional distribution in ARCH-type models. McAleer [28] provides a comprehensive comparison of a wide range of univariate and multivariate, conditional and stochastic volatility models.

Bollerslev [7] uses the Student-t distribution, while Nelson [31] uses the generalized error distribution (GED). Bollerslev et al. [8] and Watanabe [36] have found that the Student-t distribution is adequate for capturing the excess kurtosis of the conditional distribution for daily US and Japanese stock returns, respectively. Recently, Bai et al. [5] found that a mixture of two normal distributions gives a better fit to the IBM daily returns and exchange rate returns of the Dollar/Deutschemark than the Student-t distribution.

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In the literature of SV models, Liesenfeld and Jung [26] fitted a Student-t distribution and a GED as well as a normal distribution to the error distribution in the SV model, by using the simulated maximum likelihood method developed by Danielsson and Richard [15] and Danielsson [14].

In addition to a normal distribution, a Student-t distribution and a GED, the current paper considers a mixture-of-normal distributions as the error distribution in the SV model and compares which distribution is the most adequate. In order to estimate these models, the paper extends Bayesian method introduced by Jacquier et al. [22] and developed by Shephard and Pitt [33]. Specifically, both of the model parameters and the latent volatility are sampled from their posterior distribution using Markov-chain Monte Carlo (MCMC) techniques, and simulated draws are used for Bayesian posterior analysis. The latent volatility are sampled using the multi-move sampler proposed by Shephard and Pitt [33] to improve the convergence rate of the MCMC. Bayes factors are calculated by using the method proposed by Chib [9], Chib and Jeliazkov [11,12] to compare the fit of distributions.

Using the Bayesian MCMC method, the SV models with four kinds of distributions are fitted to daily data from the Yen/Dollar exchange rate (YEN/USD) and the TOPIX. According to Bayes factors, we find that the mixture-of-normal distributions and the Student-t distribution fit both data series better than the normal and the GED. While the mixture-of-normal distributions gives a better fit to the YEN/USD returns, the Student-t distributions does to TOPIX returns. The paper also examine how the specification of error distributions influences the autocorrelation functions of squared returns, the confidence intervals of future returns, and Bayes factors compared to GARCH specifications.

The rest of the paper is organized as follows. Section 2 shortly explains the SV model with four distributions mentioned above. Section 3 develops the Bayesian method for the analysis of the SV models with these distributions. These SV models are fitted to daily data from the YEN/USD and the TOPIX in Section 4. Conclusions are given in Section 5.

2. SV models with heavy-tailed distributions

The SV model analyzed in this article is the standard one given by

$$r_t = \epsilon_t \exp(h_t/2), \quad \epsilon_t \sim i.i.d., E(\epsilon_t) = 0,\ V(\epsilon_t) = 1, \tag{1}$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \eta_t, \quad \eta_t \sim i.i.d. N(0, \sigma_\eta^2), \tag{2}$$

where $|\phi| < 1$, and r_t is the asset return on day t from which the mean and autocorrelations are removed. In the following, $\exp(h_t/2)$ is called as volatility, so that h_t represents the log of squared volatility. ϵ_t and η_s are assumed to be serially and mutually independent for all t and s.

It is a well-known phenomenon that daily asset returns have leptokurtic distributions. The kurtosis of r_t following the above SV model is given by $k = E(r_t^4)/E(r_t^2)^2 = E(\epsilon_t^4)\exp[\sigma_h^2]$, where $\sigma_h^2 = \sigma_\eta^2/(1-\phi^2)$ represents the unconditional variance of h_t; see Appendix A in Liesenfeld and Jung [26] for the derivation.

The standard normal distribution is usually assumed for the distribution of ϵ_t. If so, $E(\epsilon_t^4) = 3$ and hence the kurtosis of r_t is $k = 3\exp[\sigma_h^2] \geq 3$, where $k = 3$ only if $\sigma_h^2 = 0$. This result indicates that, as long as the volatility changes over time, the unconditional distribution of r_t is leptokurtic even if ϵ_t follows the standard normal. However, it does not necessarily follow that the leptokurtosis of asset returns can fully be explained by changes in volatility. The distribution of ϵ_t itself may possibly be leptokurtic.

The paper considers a mixture of two normal distributions as follows;

$$\epsilon_t \sim i.i.d. \text{ mixture normal}(\xi, p) = \begin{cases} N(0, \sigma^2) & \text{with probability } (1-p), \\ N(0, \xi\sigma^2) & \text{with probability } p, \end{cases}$$

where $0 < \xi < 1$ and $\sigma^2 = (1 - p + \xi p)^{-1}$ so that $\mathrm{Var}(\epsilon_t^2) = 1$. The probability density function (PDF) of ϵ_t is given by

$$f(\epsilon_t) = \frac{p}{\sqrt{2\pi\xi\sigma^2}} \exp\left(-\frac{\epsilon_t^2}{2\xi\sigma^2}\right) + \frac{1-p}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\epsilon_t^2}{2\sigma^2}\right). \tag{3}$$

The kurtosis of the mixture-of-normal distributions is given by $E(\epsilon_t^4) = 3 + 3p(1-p)(\xi-1)^2/(1-p+\xi p)^2$, which is greater than 3 when $0 < p < 1$.