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Inference for Weibull distribution under generalized order statistics

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Abstract

Based on generalized order statistics from Weibull distribution the approach of Bayesian and non-Bayesian estimation are discussed. We present a simple and efficient simulational algorithm for generating a generalized order statistics sample from any continuous distribution. Specializations to Bayesian and non-Bayesian estimators, some lifetime parameters and confidence intervals of progressive II censoring and record values are obtained and compared with the existing results. Two examples are given to illustrate the proposed estimators and the simulation algorithm.

Keywords: Bayesian estimator; Symmetric loss function; Generalized order statistics; Progressive censoring; Record values

1. Introduction

A concept of generalized order statistics (gos) was introduced by Ref. [15], as a general framework for models of ordered random variables. Moreover, many other models of ordered random variables, e.g., ordinary order statistics (os), *k*-record values and progressively Type II censoring, are seen to be particular cases of gos. These models can be effectively applied, e.g., in reliability theory.

Refs. [3] and [12] studied some distributional properties of gos and obtained minimum variance linear unbiased estimates of the parameters of exponential and Pareto type II distributions based on gos, respectively. Ref. [19] obtained maximum likelihood estimators (MLEs) for the parameters of Burr XII distributions based on gos. Bayesian predictive densities and survival functions of gos studied in Ref. [5]. Recently Ref. [1] obtained the Fisher information in an order statistics and its concomitants based on gos.

Recall the concept of gos (see [15]). Let F be an absolutely continuous distribution function with density function f. Let $n \in N$, $m = (m_1, ..., m_{n-1}) \in R_{n-1}$, $k > 0$, be given constants such that for all $1 \le i \le n-1$, $\gamma_i = k + n - i + M_i > 0$, where $M_i = \sum_{j=i}^{n-1} m_j$. The random variables $X(r, n, \breve{m}, k)$, $1 \le r \le n$, are said to be gos if their joint density function

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is of the form (see [15])

$$f_{1,2,\cdots,n}(x_1, x_2, \cdots, x_n) = \begin{cases} k\{\prod_{j=1}^{n-1}\gamma_j\}\prod_{j=1}^{n-1}\bar{F}^{m_j}(x_j)f_1(x_j)\bar{F}^{k-1}(x_n)f(x_n), & F^{-1}(0) < x_1 < x_2 <, \cdots < x_n < F^{-1}(1) \\ 0, & \text{otherwise,} \end{cases} \tag{1.1}$$

where $\bar{F}(x) = 1 - F(x)$.

The Weibull distribution has been used very effectively for analyzing lifetime data, particulary when the data are censored, which is very common in most life testing experiments. Among the different censoring schemes, the progressive censoring scheme and record values have received a considerable attention in the last few years, see Refs. [6] and [4]. The pdf of Weibuul distribution is given by

$$f(y; \theta, \beta) = \frac{\beta}{\theta} y^{\beta-1} \exp - \left(\frac{y^\beta}{\theta}\right), y > 0; \beta > 0; \theta > 0, \tag{1.2}$$

where θ is the scale parameter and β is the shape parameter.

The reliability function $R(t)$, and the hazard function $H(t)$ at time t are given by

$$R(t) = \exp\left(-\left(\frac{t^\beta}{\theta}\right)\right), t > 0, \tag{1.3}$$

and

$$H(t) = \frac{\beta}{\theta} t^{\beta-1}, t > 0. \tag{1.4}$$

Several authors have Bayesian estimators and MLEs based on progressive censoring scheme. Among others are Refs. [7,2,11,13,21,23]. Also based on record values, Bayesian estimators and MLEs were discussed by many authors for example, see Refs. [25,26,14] and the references therein.

But up to now, Bayesian and non-Bayesian estimation related to Weibull distribution were not addressed under the gos models. So in this paper Bayesian estimators under squared error loss (sel) function and MLEs for the parameters of Weibull distribution are derived based on gos. At the same time we give Bayesian estimators and MLEs for the hazard and reliability functions and highest posterior density (HPD) credible interval for the scale parameter. In addition, we give a simple and efficient simulational algorithm for generating a gos sample from any continuous distribution. Specializations to Bayesian estimators and MLEs of progressive II censoring and record values are obtained and compared with the existing results. Finally we provides two examples, the first example using real survival data, for illustration. The second example using simulated data to illustrate the presented algorithm.

2. Prior, posterior and Bayes estimators

Suppose that $X_{1,n,\tilde{m},k}, X_{2,n,\tilde{m},k}, \cdots, X_{n,n,\tilde{m},k}, k \geq 1$, are n gos based on the df from Weibull distribution. According to (1.2) and (1.1), the likelihood function is

$$L(\beta, \theta; \mathbf{x}) = k\left(\frac{\beta}{\theta}\right)^n \left\{\prod_{j=1}^{n-1}\gamma_j\right\} u^{\beta-1} \exp\left[-\left(\frac{z}{\theta}\right)\right], \tag{2.1}$$

where $z = \sum_{j=1}^{n-1}(m_j + 1)x_j^\beta + kx_n^\beta$ and $u = \prod_{j=1}^{n-1}x_j$.