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Modified procedures for change point monitoring in linear models

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Abstract

Monitoring on-line data to detect change point as early as possible is an important issue. It is shown that the existing CUSUM test is inefficient to quickly give an alarm when change point does not occur at the early stage of monitoring. In this paper we propose a set of new monitoring procedures to detect coefficients and error variance change in linear regression models. Our proposed modification, which uses a bandwidth parameter to change the beginning time of monitoring, can detect change point more quickly even if it occurs after a relative longer monitoring time. Simulations suggest that the modified procedures compared with the CUSUM test have the same null distribution but higher power and shorter average run length. In particular, we illustrate the effectiveness of our procedures by IBM stock data and Thailand/U.S. foreign exchange rate data.

Keywords: Linear model; Change point monitoring; Bandwidth; Average run length

1. Introduction

Since the seminal work of Chu et al. [5], change point monitoring became a popular topic among economists and statisticians. The idea of Chu et al. [5] is based on a fixed historical sample, one can monitor the new observed samples sequentially for any sample size when there is no change and give an alarm quickly when change point occurs. According to this idea, Andreou and Ghysels [1], Zeileis et al. [16] studied structure change monitoring problem for dynamic econometric models. Fukuda [7] tested the performance of unit root monitoring by simulation. Aue et al. [3] and Horváth et al. [9] proposed a monitoring scheme for linear regression model according to the OLS residuals and boundary function

$$g(m,k) = m^{1/2}\left(1+\frac{k}{m}\right)\left(\frac{k}{m+k}\right)^{\gamma}, \quad 0 \le \gamma < \frac{1}{2}.$$

More recently, Horváth et al. [11] studied the case for $\gamma = 1/2$. Simulations indicate that their procedure is very sensitive for changes which occur shortly after the training period when choose γ near to 1/2. However, the procedure

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is insensitive for changes that don't occur at the early stage of monitoring. In other words, their procedure has lower power and longer average run length (ARL) when change point occurs after a relative longer monitoring time. If we fail to find change point quickly in practice, one may suffer much risk. Hence, studying for a monitoring scheme, which will still sensitive for changes occurring after a relative longer monitoring time, is necessary and meaningful.

With the above considerations in mind, we propose in this paper a modified monitoring scheme to make it still sensitive even if change point occurs after a longer monitoring time. In the modified procedure, we add a parameter h in the traditional monitoring function and boundary function to improve the monitoring power and ARL. The idea to introduce this parameter is "moving" change point (if occurs) to earlier stage of monitoring by changing the beginning time of monitoring. Simulations indicate that the modified procedure has the same null distribution as Horváth et al. [9], but has higher power and shorter ARL.

Since the variance change is often interpreted as a risk in econometrics, detecting variance change also is an important issue in time series analysis. Many papers have considered this problem, for example, Csörgő and Horváth [6], Inclán and Tiao [12], Lee et al. [13–15], among many others. However, all these papers concentrate on retrospective test. For sequential variance change point monitoring problem, Carsoule and Franses [4] monitored normal random variable series and ARCH(1) process, Horváth et al. [10] investigated change of unconditional variance in conditionally heteroskedastic time series. Following the above modification idea, we also monitor error variance change in linear regression models. For this problem we use residual CUSUM of squares to construct monitoring function and obtain the same null distribution as the coefficients change problem case. Simulations indicate that this procedure is still sensitive for coefficients change, but the coefficients change monitoring procedure has poor performance for variance change. So we can use these two procedures to distinguish coefficients change and variance change.

The rest of the paper is organized as follows. Section 2 specifies the monitoring problem and shows all necessary assumptions. Section 3 contains the modified monitoring procedures and their asymptotics. In Section 4, we first compare the size, power and ARL performance of our monitoring procedures with literatures' by Monte Carlo method based on normal disturbances, and then reports the simulations of two monitoring procedures under GARCH(1,1) error case. Two real data examples are also given in this section. Section 5 concludes the paper. All proofs of the theorems are gathered in the Section 6.

2. Testing problem and assumptions

Consider the same linear regression model as Horváth et al. [9].

$$y_i = \mathbf{x}_i^{\mathrm{T}}\beta_i + \varepsilon_i, \quad 1 \le i < \infty, \tag{1}$$

where $\mathbf{x}_i$ is a $p \times 1$ dimensional random vector of the form $\mathbf{x}_i^{\mathrm{T}} = (1, x_{2,1}, \ldots, x_{p,i})$, β_i is a $p \times 1$ dimensional random parameter vector and $\{\varepsilon_i\}$ is an error sequence with $E\varepsilon_i = 0$, and $E\varepsilon_i^2 = \sigma_i^2$.

First, we state the assumptions which are needed to prove the asymptotic validity of our procedure.

Assumption 2.1. There is a positive definite matrix $\mathbf{C}$ and a constant $\tau > 0$ such that

$$\left| \frac{1}{n}\sum_{i=1}^{n}\mathbf{x}_i\mathbf{x}_i^{\mathrm{T}} - \mathbf{C} \right| = O(n^{-\tau}), \quad \text{a.s.} \quad (n \to \infty) \tag{2}$$

and that

$$\{\varepsilon_i, 1 \le i < \infty\} \quad \text{and} \quad \{\mathbf{x}_i\} \quad \text{are independent.} \tag{3}$$

Assumption 2.2. For each m, there are independent Wiener processes $\{W_{1,m}(t) : t \ge 0\}$ and $\{W_{2,m}(t) : t \ge 0\}$ and a constant $\sigma > 0$ such that

$$\sup_{1 \le k < \infty} \frac{1}{k^{\lambda}} \left| \sum_{i=m+1}^{m+k} \varepsilon_i - \sigma W_{1,m}(k) \right| = O_p(1) \quad (m \to \infty) \tag{4}$$