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Domain decomposition methods for the neutron diffusion problem

Pierre Guérin [a,*], Anne-Marie Baudron [b], Jean-Jacques Lautard [b]

[a] *EDF R&D, 1 avenue du Général De Gaulle, 92141 Clamart Cedex, France*
[b] *Commissariat à l'Energie Atomique, DEN/DANS/DM2S/SERMA/LLPR, CEA Saclay, 91191 Gif-sur-Yvette, France*

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Abstract

The neutronic simulation of a nuclear reactor core is performed using the neutron transport equation, and leads to an eigenvalue problem in the steady-state case. Among the deterministic resolution methods, simplified transport (SP_N) or diffusion approximations are often used. The MINOS solver developed at CEA Saclay uses a mixed dual finite element method for the resolution of these problems, and has shown his efficiency. In order to take into account the heterogeneities of the geometry, a very fine mesh is generally required, and leads to expensive calculations for industrial applications. In order to take advantage of parallel computers, and to reduce the computing time and the local memory requirement, we propose here two domain decomposition methods based on the MINOS solver. The first approach is a component mode synthesis method on overlapping subdomains: several eigenmodes solutions of a local problem on each subdomain are taken as basis functions used for the resolution of the global problem on the whole domain. The second approach is an iterative method based on a non-overlapping domain decomposition with Robin interface conditions. At each iteration, we solve the problem on each subdomain with the interface conditions given by the solutions on the adjacent subdomains estimated at the previous iteration. Numerical results on parallel computers are presented for the diffusion model on realistic 2D and 3D cores.

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Keywords: Domain decomposition; Eigenvalue problem; Parallel computation; Diffusion equation

1. Introduction

In order to ensure the design, the security and the maintenance of a nuclear reactor core, the use of numerical simulation is necessary. The neutron balance is given by the neutron transport equation and leads to an eigenvalue problem in the steady-state case. Its resolution on 3D heterogeneous geometries is currently too expensive for industrial applications, even if simplified models are used, such as the simplified transport (SP_N) or the diffusion approximation.

A way to reduce the computing time and the local memory requirement is to use a domain decomposition method. It is particularly well fitted for parallel computers: calculations are distributed on several subdomains, and as many processors as subdomains can be used. We propose here two approaches based on domain decomposition.

The first one is a component mode synthesis approximation, and has been already presented in [6]: the global flux is expanded on a finite set of basis functions obtained on overlapping subdomains. The global exact problem is solved in the finite spaces spanned by the different local functions. Two techniques are presented in order to obtain these basis

* Corresponding author. Tel.: +33 1 47 65 36 77.
E-mail addresses: pierre.guerin@edf.fr (P. Guérin), anne-marie.baudron@cea.fr (A.-M. Baudron), jean-jacques.lautard@cea.fr (J.-J. Lautard).

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functions. The second approach is an iterative domain decomposition method using non overlapping subdomains and Robin interface conditions.

Even if these methods could be applied to the SP_N equations, we present here their application to the critical neutron diffusion problem. They are implemented in the framework of the existing MINOS solver (developed at CEA Saclay), which uses a mixed dual finite element method for the resolution of diffusion and SP_N equations in 3D cartesian geometries. We present results for these two methods on realistic 2D and 3D cores, and we show their efficiency on parallel computers.

The paper is organized as follows. In Section 2, we introduce the neutronic equations and the MINOS solver. In Section 3, we describe the component mode synthesis method applied to the steady-state neutronic equations, and in Section 4 we explain the iterative domain decomposition method. Finally we give in Section 5 conclusions and perspectives for these methods.

2. The neutronic equations and the MINOS solver

2.1. The steady-state diffusion problem

In a nuclear reactor core, the neutron balance is given by the neutron transport equation. In the steady-state case, it leads to an eigenvalue problem, due to the fission chain reaction. This problem is too expensive to be solved on an entire core: some approximations are usually made. The first one consists in the discretization of the neutron kinetic energy variable: the neutrons are gathered in a finite number of energy groups. A second approximation is the use of the diffusion model, which does not require angular variables. For more details on the neutronic equations, we refer to [11].

If R is a bounded domain (in fact the core) with boundary ∂R, the steady-state diffusion problem is a generalized eigenvalue problem, and its mixed (flux-current) formulation reads as follows for each energy group, with zero flux boundary conditions:

$$\begin{cases} \overrightarrow{p} + D\overrightarrow{\nabla}\varphi = 0 & \text{on} \quad R, \\ \overrightarrow{\nabla}\overrightarrow{p} + \sigma\varphi = \dfrac{1}{\lambda}S_f + S_\varphi & \text{on} \quad R, \\ \varphi = 0 & \text{on} \quad \partial R, \end{cases} \tag{1}$$

where $\overrightarrow{p}$ is the current, φ is the scalar flux; D is the diffusion coefficient, σ is the removal cross section. S_f is the fission source and S_φ is the scattering source, both due to the contribution of the other energy groups. The greatest eigenvalue is the multiplication factor (k_{eff}) of the chain reaction. The reaction is said to be "sub-critical" if $k_{eff} < 1$, "super-critical" if $k_{eff} > 1$ and critical if $k_{eff} = 1$. In the neutronic framework, only the fundamental mode associated to this k_{eff} has a physical sense. Only this fundamental solution is considered in the sequel.

The dual variational formulation of the problem (1) is obtained by projecting the first equation of (1) on the space $H(div, R)$, the second one on $L^2(R)$, and by applying the Green's formula to the first equation. We obtain the following variational problem for each energy group: find the fundamental eigenmode $\overrightarrow{p} \in H(div, R)$, $\varphi \in L^2(R)$ solution of the problem

$$\begin{cases} \displaystyle\int_R -\frac{1}{D}\overrightarrow{p}\cdot\overrightarrow{q} + \int_R \overrightarrow{\nabla}\overrightarrow{q}\,\varphi = 0 & \forall \overrightarrow{q} \in H(div, R), \\ \displaystyle\int_R \overrightarrow{\nabla}\overrightarrow{p}\,\psi + \int_R \sigma\varphi\psi = \frac{1}{\lambda}\int_R S_f\psi + \int_R S_\varphi\psi & \forall \psi \in L^2(R), \end{cases} \tag{2}$$

where $H(div, R) = \{\overrightarrow{q} \in L^2(R)^S, \overrightarrow{\nabla}\overrightarrow{q} \in L^2(R)\}$ with S the space dimension. For more details on variational formulations and spaces, we refer to [3].