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Calculating near-singular eigenvalues of the neutron transport operator with arbitrary order anisotropic scattering

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Abstract

The homogeneous one-speed neutron transport equation in plane geometry can be solved using a separation of variables technique. In the case of isotropic scattering and a sub-critical system, it is well known [15,5] that there are exactly two discrete eigenvalues with opposite sign associated to this separation procedure. In the case of arbitrary order anisotropic scattering, there can be many more of those plus–minus pairs of discrete eigenvalues. When in a subsequent step, the corresponding eigenfunctions are used as a basis set for the expansion of a general solution to the neutron transport equation, it is of utmost importance to be able to find all discrete eigenvalues in order to have a complete set. In this paper we briefly describe the three step procedure we have developed to calculate all discrete eigenvalues. During numerical tests with this procedure, we found that there exist cases where there is a discrete eigenvalue located extremely close to the singular point at unity. The main part of this paper is devoted to the description of how we needed to modify our routines in order to calculate these so-called near-singular eigenvalues. Our most important boundary condition was that we did not wish to resort to high-precision fixed point arithmetic but would solely rely on IEEE 754 double precision arithmetic.

Keywords: Neutron transport; Anisotropic scattering; Near-singular eigenvalues

1. Introduction

The homogeneous one-speed neutron transport equation in plane geometry can be solved using a separation of variables technique. In the case of isotropic scattering and a sub-critical system, it is well known that there are exactly two discrete eigenvalues with opposite sign associated to this separation procedure. Many systems and materials, however, do not scatter the neutrons equiprobable in every direction but have a preferential direction. Typically, light elements show a preference for backward scattering while heavy elements scatter in the forward direction.

In the case of arbitrary order anisotropic scattering, there can be many more of those plus–minus pairs of discrete eigenvalues. When in a subsequent step, the corresponding eigenfunctions are used as a basis set for the expansion of a general solution to the neutron transport equation, it is of utmost importance to be able to find all discrete eigenvalues in order to have a complete set.

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Anisotropic scattering becomes very important in different applications of transport phenomena, for example in nuclear reactor core calculations or the simulation of light traveling through clouds. Our motivation for this work was the core calculations for the Accelerator Driven System (ADS) MYRRHA currently under development at SCK·CEN [1].

The developments described in this paper are part of the development of our plane transport boundary source method for solving the neutron transport equation for piecewise homogeneous media that can have an arbitrary order anisotropic scattering [23]. The code PTASBSM is completely written in C++ and relies solely on IEEE 754 double precision arithmetic.

2. The discrete spectrum of the neutron transport operator

The homogeneous one-speed neutron transport equation in plane geometry can be written

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \Sigma_t \psi(x,\mu) = \frac{c\Sigma_t}{2} \sum_{l=0}^{N} (2l+1) f_l P_l(\mu) \int_{-1}^{+1} \psi(x,\mu')\, P_l(\mu')\, \mathrm{d}\mu'. \tag{1}$$

where we have used the classical notation: x is the spatial coordinate; μ is the angular coordinate; Σ_t is the total macroscopic cross section; c is the number of secondary neutrons per collision; f_l are the scattering coefficients up to order N and $P_l(\mu)$ is the Legendre polynomial of degree l.

We assume the scattering function $f(\mu) = \sum_{l=0}^{N} \frac{2l+1}{2} f_l P_l(\mu)$ to be non-negative on $\mu \in [-1, +1]$ and normalized so that $f_0 = \int_{-1}^{+1} f(\mu)\, \mathrm{d}\mu = 1$. Letting $\psi(x, \mu) = X(x)\phi(\mu)$, the classical approach to solving this equation by separation of variables leads to

$$-\frac{1}{\Sigma_t X(x)} \frac{dX(x)}{dx} = \frac{1}{\phi(\mu)} B\phi(\mu) = \text{cte} \tag{2}$$

where

$$B\phi(\mu) = \frac{1}{\mu}\phi(\mu) - \frac{c}{2\mu} \sum_{l=0}^{N} (2l+1) f_l P_l(\mu) \int_{-1}^{+1} \phi(\mu') P_l(\mu')\, \mathrm{d}\mu'. \tag{3}$$

Using the standard convention to denote by $1/\nu$ the separation constant, one can index the corresponding spatial and angular components by ν:

$$\frac{\mathrm{d}X_\nu(x)}{\mathrm{d}x} = -\frac{\Sigma_t}{\nu} X_\nu(x) \tag{4}$$

and

$$B\phi_\nu(\mu) = \frac{1}{\nu}\phi_\nu(\mu). \tag{5}$$

By using the normalization $X_\nu(x)$ by $X_\nu(0) = 1$, the Eq. (4) gives

$$X_\nu(x) = e^{-\frac{\Sigma_t x}{\nu}} \tag{6}$$

The $\phi_\nu(\mu)$ are normalized by $\int_{-1}^{+1} \phi_\nu(\mu)\, \mathrm{d}\mu = 1$. Using both normalizations and after some manipulations [5], the Eq. (5) can be rewritten as

$$\phi_\nu(\mu) = \frac{c\nu}{2} \frac{1}{\nu - \mu} \sum_{l=0}^{N} (2l+1)\, f_l\, g_l(\nu)\, P_l(\mu) \tag{7}$$

where $g_l(\nu)$ denote the so-called Chandrasekhar or transport polynomials defined by the recurrence

$$(2l+1)\nu(1 - cf_l)g_l(\nu) = (l+1)g_{l+1}(\nu) + lg_{l-1}(\nu) \tag{8}$$

with $g_0(\nu) = 1$ and $g_1(\nu) = \nu(1 - c)$ (which turn to be equal to the Legendre moments of $\phi_\nu(\mu)$). Finally, inserting (7) into the normalization condition for $\phi_\nu(\mu)$ gives rise to the so-called dispersion relation or characteristic