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Original Article

Plant disease models with nonlinear impulsive cultural control strategies for vegetatively propagated plants

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Abstract

In present work, with aims to eradicate plant diseases or maintain the number of infected plants below the Economic Threshold, the plant disease models including nonlinear impulsive functions and cultural control strategies are proposed and analyzed. Firstly, the model with impulsive effects at fixed moments is considered. The existence and stability of the disease-free periodic solution of the model are investigated. Moreover, we establish conditions for the permanence of the system and obtain the sufficient conditions under which the positive periodic solution exists by bifurcation theory. Secondly, the impulsive model with Economic Threshold for the infected plants is analyzed. The existence and stability of the positive periodic solution are discussed. Further, the complete expression of the period of the periodic solution is obtained. Our main results imply that we can choose proper control frequency and intensity to either eradicate the plant disease or maintain the number of infected plants below the Economic Threshold. The modeling and analytic methods presented here extend the classical results for the systems with impulsive interventions, and the findings can serve as an integrating measure to design appropriate plant disease control strategies.

Keywords: Plant disease; Integrated disease management; Persistence; Economic Threshold

1. Introduction

Plant diseases are a major threat to agricultural production, which can cause severe losses in economy. According to the statistics, 14.1% crops are lost due to plant diseases alone and the total worldwide crop loss from plant diseases is about US $220 billion dollars [11]. Therefore, farmers, agro-ecological sectors and experts are very concerned about the issue of controlling plant diseases. Researchers have considered a number of dynamic processes such as the growth of plants and the spread of diseases in order to contain various diseases suffered by crops. On this basis, the integrated disease management (IDM), a long term management strategy, is developed gradually. The IDM combines several methods such as biological, cultural and chemical tactics to reduce the numbers of infected individuals to a tolerable level and aims to minimize losses and maximize returns [7,8]. Four main control strategies for vegetatively propagated plant diseases can be identified on the principle of IDM: (i) containing transmission vectors, (ii) improving the production of planting material, (iii) controlling the crop sanitation through removal of infected plants and (iv)

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breeding plants for resistance to the virus. The third one, called the cultural strategy, is widely used into practice [19,20,2,24].

Under the guidance of IDM, mathematical modeling is an increasingly valuable tool to describe, analyze and predict plant epidemics [4,25,6,21]. For instance, to determine the cost-effectiveness of the eradication procedure, an epidemiological-economic model for the citrus tristeza virus (CTV) infection and spread was developed and elaborated [4]. It was concluded that the discovery-eradication program is superior to the no-control policy from the economic viewpoint. To make wise and reasonable decisions in the practice of plant disease management, Madden et al. [12] introduced and proposed lots of mathematical and statistical models for the study of the temporal and spatial dynamics of plant disease epidemics by using the essential principles and concepts of plant disease epidemiology. Van den Bosch et al. [19] have investigated vegetatively propagated plant diseases and developed a mathematical model with continuous control strategies, which reads

$$\begin{cases} \dfrac{dS(t)}{dt} = \sigma\phi + \sigma(1-\phi)\dfrac{r(1-p)I+S}{(1-p)I+S} - \eta S - \beta SI, \\ \dfrac{dI(t)}{dt} = \sigma(1-\phi)\dfrac{(1-r)(1-p)I}{(1-p)I+S} - \eta I - \omega I + \beta SI, \end{cases} \tag{1}$$

where S and I denote the number of susceptible plants and infected plants respectively. σ represents the continual replanting rate. Crops are planted from in vitro propagated material with proportion ϕ, which is virus free, from cuttings with proportion $1-\phi$, which are taken from previous plants, or from a combination of above means. Cuttings are operated visually or by other technical methods, and crops which are confirmed to be infected will be discarded with probability p. But some plants cutting from infected ones may be healthy because the reversion causes with probability r. $1/\eta$ is the plant life cycle or harvest time. β is the transmission coefficient from susceptible plants to infected plants. This transmission is mediated by insects or other vectors. ω represents the roguing rate for the infected plants. In system (1), the authors refer to two transmission routes of plant diseases: one is via insect vectors and the other is via cuttings from infected plants used to established a new plant. The authors adopt four control strategies above and map each method onto model parameters.

A common assumption for (1) is that the control activities occur continuously. However, the control behavior usually occurs in regular pulses [23,5,26]. A more realistic treatment of plant diseases refers to periodic inspections. For example, Fishman and Marcus [5] investigated a model for the spread of citrus tristeza disease with periodic removals and mainly studied the case of two interacting populations where infection can be transmitted from one population to another. By analyzing the properties of solutions of the model, they studied the effectiveness of removal. The first aim of this study is to extend the model (1) to a more reasonable case with periodic roguing and replanting strategies, to analyze its dynamical behavior by using theories of impulsive differential equations, to identify the main factors that lead to epidemics and the effectiveness of control strategies.

An important concept in IDM is the Economic Threshold (*ET*), at which the control measures should be introduced to prevent an increasing number of infected plants from reaching the economic injury level. However, in the above model, regardless of whether the number of infected plants reaches the *ET* or not, one always exercises control periodically, which does not measure up to the standard of IDM since it is not biologically and economically desirable. Hence, only when the number of infected plants reaches the *ET*, the strategies are then implemented. The other objective of this study is to further improve the model, and propose a plant model with *ET*, which is a more realistic description of the population dynamics of plant diseases and can be used in practical decision making. Keeping the number of infected plants below ET, we would like to concentrate on the following issues: how do we utilize the impulsive control strategies to obtain the regular and sustainable development of the plants? Can we give the frequency of implementing strategies which is appropriate to prevent an intolerable build-up of diseases powerfully?

This paper is structured as follows. We study initially the plant disease models with continuous control. The existence of disease-free and positive equilibria is shown. The basic reproduction number is calculated using the next generation matrix. We also obtain the globally stable condition for the disease-free equilibrium. Secondly, we propose an impulsive system to model the process of periodic control of plant diseases. The sufficient conditions that ensure the globally asymptotic stability of the disease-free periodic solution and the permanence of the system are given. By bifurcation theory we prove that a positive periodic solution emerges via a supercritical bifurcation. Thirdly, aiming at maintaining the number of infected plants less than *ET*, we further extend our model with periodic control to include the *ET* and give