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Near-best operators based on a $\mathcal{C}^2$ quartic spline on the uniform four-directional mesh☆

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Abstract

We present some results about the construction of quasi-interpolant operators based on a special $\mathcal{C}^2$ quartic B-spline. We show that these operators, called near-best quasi-interpolants, have the best approximation order and small infinity norms. They are obtained by solving a minimization problem that admits always a solution. We give an error bound of these quasi-interpolants and we illustrate our results by a numerical example.

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1. Introduction

Let τ be the uniform triangulation of $\mathbb{R}^2$ whose set of vertices is $\mathbb{Z}^2 \cup (\mathbb{Z} + (1/2))^2$, and whose edges are parallel to the four directions $e_1 = (1, 0)$, $e_2 = (0, 1)$, $e_3 = (1, 1)$ and $e_4 = (-1, 1)$. Let $\mathbb{P}_d$ be the space of bivariate polynomials of total degree at most d, and let $\mathcal{S}_d^r(\tau)$ be the space of bivariate piecewise polynomial functions of class $\mathcal{C}^r$ on the plane and whose restrictions to each triangular cell of τ are in $\mathbb{P}_d$.

Let T be a triangle of τ and $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ be the barycentric coordinates of a point M of $\mathbb{R}^2$ relative to T. Each polynomial p in the space $\mathbb{P}_d(T)$ of polynomials defined on T has a unique representation in the Bernstein–Bézier form

$$p(M) = \sum_{i \in \triangle_d} b(i)\, B_i^d(\lambda),$$

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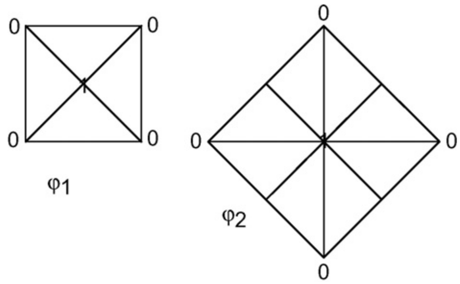

Fig. 1. B-nets and supports of φ_i for $i = 1, 2$.

where

$$\triangle_d = \{i = (i_1, i_2, i_3) \in \mathbb{Z}_+^3 : |i| = i_1 + i_2 + i_3 = d\}$$

and

$$B_i^d(\lambda) = \frac{d!}{i!}\lambda^i = \frac{d!}{i_1!i_2!i_3!}\lambda_1^{i_1}\lambda_2^{i_2}\lambda_3^{i_3}.$$

The family of the $(1/2)(d+1)(d+2)$ polynomials B_i^d, $i \in \triangle_d$, forms a basis for the space $\mathbb{P}_d(T)$. The coefficients $\{b(i), i \in \triangle_d\}$ are called the B-net of p on the triangle T.

When $r = 0$ and $d = 1$, the space $\mathcal{S}_1^0(\tau)$ of linear bivariate splines, on the four-directional mesh τ, is generated by the two minimally supported linear B-splines φ_1 and φ_2, whose B-nets and supports are given in Fig. 1.

In this paper we are interested in the space $\mathcal{S}_4^2(\tau)$ of quartic bivariate splines which is generated by three independent locally supported B-splines, denoted ψ_1, ψ_2 and ψ_3. The B-splines ψ_1 and ψ_2, constructed by Sablonnière [12], are minimally supported. The B-spline ψ_3, constructed by Chui and He [7], is quasi-minimally supported.

Using only the B-nets of these B-splines, it is not easy to show that neither these B-splines satisfy a refinement equation nor determine their associated matrix masks. That is why we introduced in [1] a new definition of these B-splines which is convenient to prove that the function vector $(\psi_1, \psi_2, \psi_3)^{\mathrm{T}}$ satisfies the refinement equation and to determine explicitly the associated refinement matrix mask. We briefly recall these results in Section 2. In Section 3, we construct spline quasi-interpolants with optimal approximation orders and small uniform norms in the space generated by a linear combination of ψ_1, ψ_2 and ψ_3, which we call near-best quasi-interpolants. They are obtained by solving a minimization problem that admits always a solution. Finally, in Section 4, we illustrate with two examples that the norms of these quasi-interpolants are small in comparison with those of the ones based on the quartic box-splines available in the literature (see e.g. [11]).

2. New definition, refinement equation and subdivision scheme

Let $\Psi := (\psi_1, \psi_2, \psi_3)^{\mathrm{T}}$ be the function vector of the above three quartic B-splines in the space $\mathcal{S}_4^2(\tau)$. We recall some results of [1], say a convolution-based definition of these B-splines and the refinement equation satisfied by Ψ.

Theorem 1. *The function vector Ψ can be expressed in terms of φ_1 and φ_2. More precisely we have*

$$\Psi^{\mathrm{T}} = (\psi_1, \psi_2, \psi_3) = (\varphi_1 * \varphi_1, 2\varphi_1 * \varphi_2, \varphi_2 * \varphi_2). \tag{1}$$

Theorem 2. *The function vector Ψ satisfies the refinement equation*

$$\Psi = \sum_{j \in \mathbb{Z}^2} P_j \;\; \Psi(2 \cdot - j), \tag{2}$$