Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 77 (2008) 209–217
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Trend analysis and computational statistical estimation in a stochastic Rayleigh model: Simulation and application

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Available online 31 August 2007

Abstract

This paper considers a stochastic model based on the homogeneous stochastic Rayleigh diffusion process. We first examine the main probabilistic characteristics of the model and describe, among other results, an explicit expression of the trends (both conditioned and nonconditioned) and, when it exists, the stationary distribution. We then obtain results of the statistical estimation of the corresponding parameters and consider the computational problems that may arise. In addition, we present an algorithm for the stochastic simulation of the sample path of the model based on the corresponding Ito stochastic differential equation. Finally, the model is applied to study the evolution of the production of thermal electricity in countries in the Maghreb region; the results obtained are in good statistical accord with the real data observed for the period 1980–2002.

Keywords: Stochastic diffusion process; Rayleigh model; Estimation; Simulation; Electricity production in the Maghreb

1991 MSC: 60J60; 62M05

1. Introduction

It is well known that in stochastic modelling an outstanding role is played by stochastic diffusion processes; these are considered either from the viewpoint of the corresponding Ito or Stratonovich stochastic differential equations (SDEs) or from the associated Kolmogorov (Fokker-Planck and backward) differential equations. Stochastic diffusions have been widely used in diverse fields, such as stochastic financial analysis, animal or cell growth in a random environment, marketing, and the natural environment. In particular, Gompertz and lognormal stochastic diffusion processes have been studied with respect to specific theoretical aspects, and they have been successfully applied to real cases in Gutiérrez et al. [12,10] and Ferrante et al. [2]. In order to apply these diffusion processes to the modelling and prediction of real phenomena, it is necessary to develop results of statistical inference, firstly on the estimation of their parameters (general results on this question can be consulted in Prakasa Rao [16]). The corresponding statistical methodology is based on a continuous sampling of the sample paths or on time-discretised observations of the dynamic variable under consideration. This may, however, run into problems related to computation and to numeric approximations with respect to the resolution of nonlinear equations and the calculation of integrals. The algorithms used to simulate the sample paths of the processes considered must also be based upon discretised approximations of the corresponding Ito

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0378-4754/$32.00 © 2007 IMACS. Published by Elsevier B.V. All rights reserved.
doi:10.1016/j.matcom.2007.08.017

stochastic equations. This methodology has been considered by several authors recently; specifically, for Gompertz and logistic lognormal diffusion processes, real results and applications are available, see for example Gutiérrez et al. [11,9] and Giovanis and Skiadas [4]. With respect to the Rayleigh process considered in the present paper, we study the above-mentioned questions and establish a methodology that enables it to be statistically fitted to real observed data. In Section 2 we consider basic theoretical results of a probabilistic nature that complement the results obtained by Giorno et al. [3], and we obtain the moment functions expressed in terms of Kummer functions, as well as the stationary distribution. In particular, we obtain the trends (the conditioned and the nonconditioned mean functions). Subsequently, in Section 3, we present results of the estimation of the parameters of the model, obtained by the continuous sampling of sample paths and by the maximum likelihood method; we then propose a methodology to resolve the computational problems that may arise in its calculation. Finally, in Section 4 we analyse the modelling procedure, using the Rayleigh model proposed, of the dynamic evolution of the trend corresponding to thermal electricity production in countries within the Maghreb region, using real data for annual production for the period 1980–2002. The variable that is considered is of great interest in environmental scientific studies concerning, the global emission of CO_2 (see Gutiérrez et al. [8]).

2. The model and its basic probabilistic characteristics

2.1. The proposed model

The stochastic model proposed is based on a homogeneous stochastic Rayleigh diffusion process (HSRDP), which is defined as a Markov process $\{X_t; t \in [0, T]\}$, with values in $]0, \infty[$, and sample paths that are almost surely continuous and with infinitesimal moments (drift and diffusion coefficient) that are given by (see Giorno et al. [3])

$$A_1(x) = \frac{a}{x} + bx, \qquad A_2(x) = \sigma^2, \tag{1}$$

where σ, a and b $(b \neq 0)$ are real parameters.

If $f(y, t|x, s)$ denotes the transition probability density function (TPDF) of this process, under certain conditions of regularity, which are satisfied by (1) and with the initial condition $\lim_{t\to s} f(y, t|x, s) = \delta(y - x)$, the above-mentioned TPDF satisfies the Kolmogorov (Fokker-Planck) forward equation

$$\frac{\partial f(y, t|x, s)}{\partial t} = -\frac{\partial[A_1(y) f(y, t|x, s)]}{\partial y} + \frac{1}{2}\frac{\partial^2[A_2(y) f(y, t|x, s)]}{\partial y^2} \tag{2}$$

Alternatively, under known conditions of regularity (see for example, Wong and Hajek [17]) satisfied by (1), the above process can be considered as the unique solution (with a probability of 1) of Ito's stochastic differential equation (SDE)

$$\mathrm{d}X_t = \left(\frac{a}{X_t} + bX_t\right)\mathrm{d}t + \sigma \mathrm{d}W_t, \quad X_0 = x_0, \tag{3}$$

where W_t is a standard Wiener process.

The unique solution (a.s.) to the above Eq. (2) can be obtained (see Giorno et al. [3]) and can be expressed as follows

$$f(x, t|y, s) = \frac{2by^{-\alpha}x^{\alpha+1}\,\mathrm{e}^{-\alpha b(t-s)}}{\sigma^2(\mathrm{e}^{2b(t-s)} - 1)} \exp\left(\frac{-b(x^2 + y^2\,\mathrm{e}^{2b(t-s)})}{\sigma^2(\mathrm{e}^{2b(t-s)} - 1)}\right) I_\alpha\left(\frac{2bxy\,\mathrm{e}^{b(t-s)}}{\sigma^2(\mathrm{e}^{2b(t-s)} - 1)}\right)$$

for $a > -\sigma^2/2$ and with the zero-flux condition. In this expression I_α denotes the modified Bessel function of the first kind and $\alpha = a/\sigma^2 - 1/2$.

2.2. Moments of the model

In Giorno et al. [3], certain probabilistic aspects of the HSRDP are ignored; nevertheless, these are of great interest in statistical modelling methodology and in the forecasting of real cases, for example in calculating moment functions and the statistical estimation of the process parameters. In the present paper, we establish in an explicit way the conditioned moment functions of order r for HSRDP and, in particular, the trend (mean) functions, both conditioned and nonconditioned, all of which are expressed by means of Kummer functions.