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Combined characteristics and finite volume methods for sediment transport and bed morphology in surface water flows

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Abstract

We propose a new numerical method for solving the equations of coupled sediment transport and bed morphology by free-surface water flows. The mathematical formulation of these models consists of the shallow water equations for the hydraulics, an advection equation for the transport of sediment species, and an Exner equation for the bedload transport. The coupled problem forms a one-dimensional hyperbolic system of conservation laws with geometric source terms. The proposed numerical method combines the method of characteristics with a finite volume discretization of the system. The combined method is simple to implement and accurately resolves the governing equations without relying on Riemann problem solvers. Numerical results are presented for several test examples on sediment transport and bed morphology by free-surface water flows.

Keywords: Shallow water equations; Sediment transport; Bed morphology; Finite volume scheme; Method of characteristics

1. Introduction

The main concern of morphodynamics is to determine the evolution of bed levels for hydrodynamic systems such as rivers, estuaries, bays and other nearshore regions where water flows interact with the bedload geometry. Examples of applications include among others, beach profile changes due to severe wave climates, seabed response to dredging procedures or imposed structures, and harbour siltation. The ability to design numerical methods able to predict the morphodynamic evolution of the coastal seabed has a clear mathematical and engineering relevances, compare [13,15,14,3] among others. In practice, morphodynamic problems involve coupling between a hydrodynamic model, which provides a description of the flow field leading to a specification of local sediment transport rates, and an equation for bed level change which expresses the conservative balance of sediment volume and its continual redistribution with time. In the current study, the hydrodynamic model is described by the shallow water equations, the suspended sediment is modelled using an advection equation accounting for deposition and erosion effects, and the transport of the bedload is modelled by the Exner equation. The coupled models form a hyperbolic system of conservation laws with source terms. It is well known that the solutions of these systems may present steep fronts and even shock discontinuities, which need to be resolved accurately in applications and often cause severe numerical difficulties, see for example [5,3].

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The object of this study is to devise a numerical approach able to accurately approximate solution to morphodynamic problems. Our aim is to develop a family of finite volume methods that incorporate techniques from the method of characteristics into the reconstruction of numerical fluxes. Our main goal is to present a class of numerical methods that are simple, easy to implement, and accurately solves the sediment transport equations without relying on Riemann problem solvers. The method has been recently investigated in [2] for numerical solution of shallow water equations on fixed bed and it is adapted in the current work for the numerical solution of morphodynamic problems. The proposed finite volume scheme belongs to the class of methods that employ only physical fluxes and averaged states in their formulations. It can be interpreted as a predictor–corrector scheme. In the corrector stage, the considered equations are integrated over an Eulerian time–space control volume whereas in predictor stage, the sediment transport equations are rewritten in a non-conservative form and integrated along the characteristics defined by the water velocity. The main features of such a finite volume scheme are on one hand, the capability to satisfy the conservation property resulting in numerical solutions free from spurious oscillations in significant morphodynamic situations, and on the other hand the achievement of strong stability for simulations of slowly varying bedload as well as rapidly varying flows containing also shocks or discontinuities. These features are verified using several test examples of the sediment transport problems. Results presented in this paper show high resolution of the proposed finite volume schemes and permit the straightforward application of the method to more complex, physically based sediment transport models.

In this paper, first the governing equations for the sediment transport problems are formulated. Thereafter, the combined finite volume characteristics method employed to solve the sediment transport problems is presented. After experiments with the proposed approach for a variety of morphodynamic examples, accuracy and efficiency of the combined characteristics and finite volume schemes are discussed. Concluding remarks end the paper.

2. Mathematical formulation

We assume that the flow is almost horizontal, the vertical component of acceleration is vanishingly small, the pressure is taken to be hydrostatic, the free-surface gravity waves are long with respect to the mean flow depth and wave amplitude, and the water-species mixture is vertically homogeneous and non-reactive. The governing equations are obtained by balancing the net inflow of mass, momentum and species through boundaries of a control volume during an infinitesimal time interval while accounting for the accumulation of mass, resultant forces and species within the control volume, compare for example [1]. Thus, the equations for mass conservation and momentum flux balance are given by

$$\begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} &= 0, \\ \frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}\left(hu^2 + \frac{1}{2}gh^2\right) &= -gh\frac{\partial B}{\partial x} - \frac{\tau_b}{\rho_w} + \frac{\tau_\omega}{\rho_w}, \end{aligned} \tag{1}$$

where u is the depth-averaged water velocity, h is the water depth, B is the bottom topography, g is the gravitational acceleration, ρ_w is the water density, τ_b and τ_ω are respectively, the bed shear stress and the shear of the blowing wind defined by the water and wind velocities as

$$\tau_b = \rho_w C_b u|u|, \quad \tau_\omega = \rho_w C_\omega \omega|\omega|, \tag{2}$$

where ω is the velocity of wind at 10 m above water surface, C_b and C_w are respectively, the bed friction coefficient and the coefficient of wind friction which may be either constant or estimated [7],

$$C_b = \frac{n_b^2}{h^{4/3}}, \quad C_w = \rho_a\,(0.75 + 0.067|\omega|) \times 10^{-3},$$

with n_b being the Manning roughness coefficient at the bed and ρ_a is the air density. The equation for mass conservation of species is modelled by

$$\frac{\partial (hc)}{\partial t} + \frac{\partial\,(huc)}{\partial x} = E - D, \tag{3}$$