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The pricing of options for securities markets with delayed response

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Abstract

The analogue of Black–Scholes formula for vanilla call option price in conditions of (B, S)-securities market with delayed response is derived. A special case of continuous-time version of GARCH is considered. The results are compared with the results of Black and Scholes.

Keywords: (B, S)-securities market; Stochastic delay differential equations; GARCH; Black–Scholes formula

1. Introduction

In the early 1970s, Black and Scholes [3] made a major breakthrough by deriving pricing formulas for vanilla options written on the stock. Their model and its extensions assume that the probability distribution of the underlying cash flow at any given future time is lognormal. This assumption is not always satisfied by real-life options as the probability distribution of an equity has a fatter left tail and thinner right tail than the lognormal distribution (see [16]), and the assumption of constant volatility σ in financial model (such as the original Black–Scholes model) is incompatible with derivatives prices observed in the market.

The above issues have been addressed and studied in several ways, such as

(i) volatility is assumed to be a deterministic function of the time: $\sigma \equiv \sigma(t)$ (see [31]);
(ii) volatility is assumed to be a function of the time and the current level of the stock price $S(t) : \sigma \equiv \sigma(t, S(t))$ (see [16]); the dynamics of the stock price satisfies the following stochastic differential equation:

$$\mathrm{d}S(t) = \mu S(t)\,\mathrm{d}t + \sigma(t, S(t))S(t)\,\mathrm{d}W_1(t),$$

where $W_1(t)$ is a standard Wiener process;
(iii) the time variation of the volatility involves an additional source of randomness represented by $W_2(t)$ and is given by

$$\mathrm{d}\sigma(t) = a(t, \sigma(t))\,\mathrm{d}t + b(t, \sigma(t))\,\mathrm{d}W_2(t),$$

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where $W_2(t)$ and $W_1(t)$ (the initial Wiener process that governes the price process) may be correlated (see [5,17]);

(iv) the volatility depends on a random parameter x such as $\sigma(t) \equiv \sigma(x(t))$, where $x(t)$ is some random process (see [13,27–29]).

In the approach (i), the volatility coefficient is independent of the current level of the underlying stochastic process $S(t)$. This is a deterministic volatility model, and the special case where σ is a constant reduces to the well-known Black–Scholes model that suggests changes in stock prices are lognormally distributed. But the empirical test by Bollerslev [4] seems to indicate otherwise. One explanation for this problem of a lognormal model is the possibility that the variance of $\log(S(t)/S(t-1))$ changes randomly. This motivated the work of Chesney and Scott [7], where the prices are analyzed for European options using the modified Black–Scholes model of foreign currency options and a random variance model. In their works the results of Hull and White [17], Scott [24] and Wiggins [30] were used in order to incorporate randomly changing variance rates.

In the approach (ii), several ways have been developed to derive the corresponding Black–Scholes formula: one can obtain the formula by using stochastic calculus and, in particular, the Ito's formula (see [23], for example). In the book by Cox and Rubinstein [8], an alternative approach was developed: the Black–Scholes formula is interpreted as the continuous-time limit of a binomial random model. A generalized volatility coefficient of the form $\sigma(t, S(t))$ is said to be *level-dependent*. Because volatility and asset price are perfectly correlated, we have only one source of randomness given by $W_1(t)$. A time and level-dependent volatility coefficient makes the arithmetic more challenging and usually precludes the existence of a closed-form solution. However, the *arbitrage argument* based on portfolio replication and a completeness of the market remain unchanged.

The situation becomes different if the volatility is influented by a second "nontradable"source of randomness. This is addressed in the approach (iii) and (iv) and one usually obtains a *stochastic volatility model*, which is general enough to include the deterministic model as a special case. The concept of stochastic volatility was introduced by Hull and White [17], and subsequent development includes the work of Wiggins [30], Johnson and Shanno [18], Scott [24], Stein and Stein [26] and Heston [14]. We also refer to Frey [11] for an excellent survey on level-dependent and stochastic volatility models. We should mention that the approach (iv) is taken by, for example, Griego and Swishchuk [13].

There is yet another approach connected with stochastic volatility, namely, uncertain volatility scenario (see [5]). This approach is based on the uncertain volatility model developed in Avelanda et al. [2], where a concrete volatility surface is selected among a candidate set of volatility surfaces. This approach addresses the sensitivity question by computing an upper bound for the value of the portfolio under arbitrary candidate volatility, and this is achieved by choosing the local volatility $\sigma(t, S(t))$ among two extremal values $\sigma_{\min}$ and $\sigma_{\max}$ such that the value of the portfolio is maximized locally.

Assumption made implicitly by Black and Scholes [3] is that the historical performance of the (B, S)-securities markets can be ignored. In particluar, the so-called efficient market hypothesis implies that all information available is already reflected in the present price of the stock and the past stock performance gives no information that can aid in predicting future perfromance. However, some statistical studies of stock prices (see [25,1]) indicate the dependence on past returns. For example, Kind et al. [20] obtained a diffusion approximation result for processes satisfying some equations with past-dependent coefficients, and they applied this result to a model of option pricing, in which the underlying asset price volatility depends on the past evolution to obtain a generalized (asymptotic) Black–Scholes formula. Hobson and Rogers [15] suggested a new class of nonconstant volatility models, which can be extended to include the aforementioned level-dependent model and share many characteristics with the stochastic volatility model. The volatility is nonconstant and can be regarded as an endogenous factor in the sense that it is defined in terms of the *past behavior* of the stock price. This is done in such a way that the price and volatility form a multi-dimensional Markov process.

Chang and Yoree [6] studied the pricing of an European contingent claim for the (B, S)-securities markets with a hereditary price structure in the sense that the rate of change of the unit price of the bond account and rate of change of the stock account S depend not only on the current unit price but also on their historical prices. The price dynamics for the bank account and that of the stock account are described by a linear functional differential equation and a linear stochastic functional differential equation, respectively. They show that the rational price for an European contingent claim is independent of the mean growth rate of the stock.