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Original article

Estimation of spatially varying open boundary conditions for a numerical internal tidal model with adjoint method

Haibo Chen [a,b], Anzhou Cao [a], Jicai Zhang [c], Chunbao Miao [a], Xianqing Lv [a,*]

[a] *Laboratory of Physical Oceanography, Ocean University of China, Qingdao 266100, China*
[b] *China Offshore Environmental Services Ltd., Qingdao 266061, China*
[c] *Laboratory of Coast and Island Development, Nanjing University, Nanjing 210093, China*

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Abstract

The adjoint data assimilation technique is applied to the estimation of the spatially varying open boundary conditions (OBCs) for a numerical internal tidal model. The spatial variation of the OBCs is realized by the so-called 'independent point scheme' (IPS): a subset is chosen as the independent points from the full set of open boundary points and the OBCs are obtained through linear interpolation of the values at the independent points. A series of ideal experiments are carried out on a real topography to further test this assimilation model, and to numerically investigate some properties of the IPS. On the basis of the numerical results, it is shown that, in most cases, the use of the IPS can indeed effectively improve the precision of the estimation of the OBCs. Furthermore, if the independent points can be arranged reasonably the improvement may be remarkable. The IPS shows us a way to improve the estimation of the OBCs for this model.

Keywords: Parameter estimation; Spatial variation; Open boundary conditions; Adjoint method; Internal tidal model

1. Introduction

In a regional ocean model, the open boundary conditions (OBCs), which must be prescribed to complete the model description at non-land boundaries, are very important and have a critical impact on the modeling results. The treatment of OBCs is an old topic and has been a concern in the regional ocean modeling for sometime. Many schemes have been proposed for different situations. Broadly, OBCs can be divided into two classes, passive and active [49]. The passive OBCs generally fall into two categories: the radiation and characteristic boundary conditions and the relaxation (sponge) conditions. The discussion on the passive boundary conditions is still one of the most interesting topics in recent years. On the other hand, the active OBCs are used to drive the simulation, cf. [3,10,30], and references therein. Marchesiello et al. [41] proposed a combination, an adaptive boundary condition, where different definitions are used depending on whether information is entering or leaving the domain. In this paper, the passive Flather condition [25] is modified to an active condition on normal velocity, by including the effect of known values of elevation and normal

* Corresponding author. Tel.: +86 532 66782971.
E-mail addresses: darling1242006@163.com (H. Chen), qzyzcaz@163.com (A. Cao), jczhang2008@yahoo.cn (J. Zhang), miaocb@sina.com (C. Miao), xqinglv@ouc.edu.cn (X. Lv).

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velocity that represent conditions beyond the model boundary (i.e. tidal force), which has also been used in quite a few studies (e.g. [10,30]).

In the active OBCs, the boundary solution is fully or partially specified by using external data at every boundary point or at boundary inflow points only. In practical ocean modeling, on the other hand, the external data can be obtained either from available observations near the open boundaries (tidal gauge data or satellite data) or, within a nested approach, from larger domain numerical models such as Schwiderski's global tidal model [56] and the TPXO.3 global tidal model [21]. Unfortunately, observations at open waters are often scarce and the global tidal model results are less accurate in shallow waters. Therefore, a major difficulty faced by regional ocean models is concerned with the treatment of the OBCs [35].

With the development of large ocean observing programs and remote sensing techniques, increasingly more oceanic data is becoming available. This provides a promising prospect for improving the precision of ocean modeling with these observations. In ocean modeling, it is common to solve the inverse problem, in which the OBCs are considered as tunable control parameters of the dynamical model and are determined, at least in part, from measurement data in the interior of the domain by optimization methods. That is, in the general context of control theory, one interchanges the roles of boundary values (no longer known) and the interior fields (partially known from observations). Control theory provides a general theoretical understanding of such problems, and in a fundamental sense, there is no doubt that such methods work very effectively [3]. A few attempts have been made previously (e.g. [43,57]). Indeed, this is an active topic of a fast-growing research field—data assimilation, which is an effective method for marine research, and has become widely used in meteorological and oceanographic predictions in recent years. Among all data assimilation methods, four-dimensional variational (4D-Var) data assimilation is one of the most effective and powerful approaches developed over the past two decades. It is an advanced data assimilation method which involves the adjoint technique using the optimization technique based upon Lagrange multipliers (e.g. [37,66]), and has the advantage of directly assimilating various observations distributed in time and space into the numerical model while maintaining dynamical and physical consistency with the model. Rodrigues et al. [54] applied an adjoint problem formulation to the simultaneous estimation of spatially dependent diffusion coefficient and source term in a one-dimensional nonlinear diffusion problem. In their approach, no a priori assumption is required regarding the functional form of the unknowns. Using an adjoint sensitive method, Tber et al. [61] studied the simultaneous identification of spatially distributed hydraulic conductivity and storativity in a seawater intrusion model. The 4D-Var data assimilation with the adjoint method has also been widely applied to the estimation of the OBCs. Early studies include Lardner et al. [35], Seiler [57], Bennett and McIntosh [8], Bennett [6], Zou et al. [75], Heemink et al. [29]; recent texts are Ayoub [3], Zhang et al. [71], Gejadze and Copeland [26], Gejadze et al. [27], Nguyen [46], Zhang and Lu [72,73], Chen et al. [14].

This study is an extension of [14] (hereafter CML2012), which constructed an adjoint assimilation model for numerical simulation of internal tides. In CML2012, the model was simply tested with a series of ideal experiments, in which several prescribed spatial distributions of OBCs were successfully inverted by assimilating the model-generated pseudo-observations. However, although the inverted OBCs were consistent with the prescribed ones very well, a considerable amount of noises (small fluctuations) are detectable in the solution. This may be caused, to some extent, by the ill-posedness of the optimization problem. As noted by Yeh and Sun [68] and Yeh [67] in the work of ground water flow parameter estimation, the inverse or parameter estimation problem is often ill-posed and beset by instability and non-uniqueness, particularly if one seeks parameters distributed in space and time domain. The same viewpoint has been put forward by Heemink et al. [29] Zhang and Lu [72], Smedstad and O'Brien [60], Das and Lardner [19,20], Lardner and Das [36], Ullman and Wilson [65], Alekseev et al. [1]. In these works they proposed to insert an additional criterion into the cost function named the penalty term, and by doing this, large fluctuations will be penalized to ensure that the parameters vary smoothly. Numerous studies indicate that the problem of primitive equations with OBCs is ill-posed in the sense that the existence of a unique physically realistic solution is not guaranteed (see [7,9,12,32,41,48]). Under some conditions the inviscid primitive equations may be well-posed, e.g. 'if boundary conditions are formulated in terms of local eigenfunction expansions or nonlocal boundary operators are used' [48]. Blayo and Debreu [9] pointed out that even well-posed model equations do not guarantee an accurate solution. Lu and Zhang [40], Zhang and Lu [72] and Zhang et al. [74] demonstrated that the use of the independent points to determine the 2-D bottom friction coefficient is effective as well as physically reasonable. Nevertheless, it is still valuable, although simple, to have an examination of the performance of this technique (called 'independent point scheme' (IPS) in this paper) when borrowed to the estimation of the spatially varying OBCs in a system where the available observations are sparse and much fewer than those in previous work of CML2012, which is the main objective of this paper.