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Original article

Stability and dynamics analysis of time delayed eutrophication ecological model based upon the Zeya reservoir

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Abstract

In this paper, within the framework of ordinary differential equation and eutrophication ecological theory, a time delayed eutrophication ecological model based upon the Zeya reservoir is studied analytically and numerically. A key advantage of time delayed term is that it can be quite accurate to describe food energy transformation mechanism, which enables a systematic and logical procedure for fitting eutrophication model to real monitoring data. Mathematical theoretical works have been pursuing the investigation of some stability conditions for the equilibria, which in turn provides a theoretical basis for the numerical simulation. Numerical analysis indicates that the time delayed term has an important role in complex dynamics using comparative analysis of the bifurcation diagram. It should also be stressed that chaos spontaneously appears with increase of a critical parameter for the system without time delayed term. In addition, it is successful to show that the time delayed term may be a critical factor for complex dynamics, which can directly suppress the emergence of chaos and promote the system to enter into a steady state. All these results are expected to be useful in the study of the dynamic complexity of eutrophication ecosystems.

MSC: 93C15; 37L10; 93D05

Keywords: Time delayed term; Eutrophication; Equilibrium point; Characteristic equation; Amplitude perturbation

1. Introduction

Eutrophication of lakes and reservoirs has been widely and intensively studied for several decades, which is a degradation process originating from introduction of nutrients from agricultural run-off and untreated industrial and urban discharges [33]. Eutrophication has been identified as a major and serious water quality management issue [10], not only in the developing countries [1,31,35], but also in the developed countries [2,15,23]. Impairment of water quality due to eutrophication can lead to a series of problems and result in losses of ecological integrity, sustainable

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development and safe use of aquatic ecosystems [7]. It is characterized by frequently recurring algal blooms and reduction in species diversity in water at all trophic levels [10].

In 1932, Bertalanffy firstly proposed a method of using mathematical model to study the biological system [36]. He pointed out that this approach is a combination of coordination, order and purpose, which in turn formed three basic ideas of studying the biological system, namely: trophic-level analysis, system perspective and dynamic view. The main aim of modeling population dynamics is to improve the understanding of the interactions between populations and their dependence on internal and external conditions [29]. Hence, the mathematical models of biological population dynamics have not only to account for the growth and interactions between populations but also for the forecast of the disasters which are caused by the plankton like blue-green algae. Early attempts began with the logistic growth, exponential growth and Cui growth (this growth can explain the relationship between population increment and limiting resources) for biological population. With the increasing effects of eutrophication and bloom of the algae, the history of mathematical modeling plankton dynamics and biological eutrophication removal process is already quite long and has been initiated by biological science. These approaches have been refined to more realistic descriptions of the development of biological natural populations, especially algae population. In recent decades, the main researches focused on how to control the eutrophication, how to predict the outbreak of algae, how to simulate the spread tendency of algae and how to develop eutrophication control strategy based on actual data. Hence, more and more researches have given an attempt to combine the forces of ecologist and mathematical engineers in an evaluation of possible control strategies for the management of eutrophication of water bodies, which is becoming an urgent water-management problem of drinking water reservoirs [18]. Malmaeus and Hakanson have depicted the development of a lake eutrophication model, they pointed out that all changes that provided significant differences and improved predictions have been adopted [30]. Chen et al. said that their mathematical description provides a different insight how to use nitrogen and phosphorus more effectively in the eutrophication control and elimination of blue-green blooms [6].

Zeya reservoir of Wenzhou is located in subtropical regions. Because of eutrophication, the nuisance algal blooms come forth frequently in the Zeya reservoir, which caused clogging and blocking of the filtration system and resulted in millions of people without a drinking water. Eutrophication removal is achieved by two major processes, physico-chemical and biological treatment techniques, especially biological eutrophication removal from drinking water in the lake and reservoir is usually considered to accomplish optimal and economic eutrophication treatment [24]. In order to understand and improve the performance of biological eutrophication removal treatment, it is necessary to briefly to look into the mechanisms of eutrophication removal. The removal process of eutrophication in the Zeya reservoir is complex and dynamic with many variables. Since successful management of eutrophication in Zeya reservoir is often dependent on proper identification of the most likely growth limiting amount for algae, special emphasis is given to the limiting amount concept. In order to apply the biological principle to control eutrophication in the Zeya reservoir, we bring up two species of filter-feeding fish: silver carp (*Hypophthalmichthys molitrix*) and bighead carp (*Aristichthys nobilis*) [37], which are thought to be suited to control algal biomass directly in freshwater reservoir, especially cyanobacteria.

The researches on the time-delay problem have been one of the most challenging issues for the existence of time-delayed is an important source to make system instable. Time-delays are often presented in various engineering systems such as biological, economical systems, chemical processe. Time-delays are described by differential-difference equations which belong to a class of functional differential equations [21]. Some models with time delayed term in diverse biological models are extensively studied by some scholars [3,5,9,11,13,16,17,19,20,25–28,34,38–40].

The Beddington–DeAngelis functional response is introduced by Beddington [4] and DeAngelis et al. [8], independently. The main difference of this functional response from other functional responses is that it contains an extra term presenting mutual interference by predators. Although a direct link between the filter-feeding fish and algae can not be established unless quantitative methods are used, the precious works clearly show that the amount of two species are often related, and a change in one species can cause a change in the other, especially filter-feeding fish. Thus we apply Beddington–DeAngelis functional response to describe their relationship between filter-feeding fish and algae with sufficient accuracy in this paper.

Based on the above discussion and modeling is playing an increasing role in helping to identify causes and effective control measures through what if studies, and strategies are being identified for the management and control of eutrophication [12,32], the paper considers an eutrophication mathematical model