Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation 97 (2014) 234–259

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Original Article

Numerical modelling of three-phase immiscible flow in heterogeneous porous media with gravitational effects

Eduardo Abreu *

University of Campinas (UNICAMP), Department of Applied Mathematics, IMECC, CP 6065, 13083-970 Campinas, SP, Brazil

Received 5 February 2012; received in revised form 14 August 2013; accepted 15 September 2013
Available online 20 October 2013

Abstract

This paper presents a new numerical formulation for the simulation of immiscible and incompressible three-phase water–gas–oil flows in heterogeneous porous media. We take into account the gravitational effects, both variable permeability and porosity of porous medium, and explicit spatially varying capillary pressure, in the diffusive fluxes, and explicit spatially varying flux functions, in the hyperbolic operator. The new formulation is a sequential time marching fractional-step procedure based in a splitting technique to decouple the equations with mixed discretization techniques for each of the subproblems: convection, diffusion, and pressure–velocity. The system of nonlinear hyperbolic equations that models the convective transport of the fluid phases is approximated by a modified central scheme to take into account the explicit spatially discontinuous flux functions and the effects of spatially variable porosity. This scheme is coupled with a locally conservative mixed finite element formulation for solving parabolic and elliptic problems, associated respectively with the diffusive transport of fluid phases and the pressure–velocity problem. The time discretization of the parabolic problem is performed by means of an implicit backward Euler procedure. The hybrid-mixed formulation reported here is designed to handle discontinuous capillary pressures. The new method is used to numerically investigate the question of existence, and structurally stable, of three-phase flow solutions for immiscible displacements in heterogeneous porous media with gravitational effects. Our findings appear to be consistent with theoretical and experimental results available in the literature.

MSC: 76T30; 68U20

Keywords: Three-phase flow; Nonlinear convection-diffusion system; Fractional time-step method; Gravity; Nonclassical wave

1. Introduction

The purposes of this work are twofold: (a) to describe in detail a numerical procedure for solving immiscible three-phase water–gas–oil flow problems and (b) to show the numerical evidence of structurally stable nonclassical waves in porous media involving the gravity effects. In this work we are concerned in dynamic fluid flow processes in heterogeneous reservoirs where both the convective flux and diffusion functions have a spatial discontinuity. Three-phase flow in a porous medium can be modeled using an extension of Darcy's law in terms of the relative permeability

* Tel.: +55 21 35215964.
E-mail address: eabreu@ime.unicamp.br

0378-4754/$36.00 © 2013 IMACS. Published by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.matcom.2013.09.010

and capillary pressure functions of the fluid phases. Distinct empirical models have been proposed for the relative permeability expressions in petroleum engineering [22,29,43,61,62]. It is well known that for some of the models [22,61,62] the 2×2 system of conservation laws that arises when capillarity effects are neglected fails to be strictly hyperbolic somewhere in the interior of the phase space [51,58,60,64]. As a consequence, the loss of hyperbolicity can lead to the existence of nonclassical shocks (also called transitional shocks [40] or undercompressive shocks [58]) in the solutions of the three-phase incompressible flow problems [51,60].

In general, under reasonable physical assumptions about a model for immiscible three-phase flow, singularities such as umbilic points and elliptic regions are a necessary consequence of Buckley–Leverett behavior on each two-phase edge of the saturation phase space [60–62,43]. To be specific, we point out paper [11] of Marchesin et al. (2010), which presents a recent survey of aspects of the general theory of conservation laws that bear on the construction of immiscible three-phase solutions in the petroleum engineering literature. It had shown [11] that for several injection problems, solutions for Corey's model are very similar to those for Stone's model, despite the presence of an elliptic region in the latter; and they are very different from those for the Juanes–Patzek model [43], which preserves strict hyperbolicity. Indeed, in papers [51,11] were also addressed to the question of the physical existence of nonclassical waves in actual three-phase flows as we considered in this work. Additionally to the question of hyperbolicity over the whole saturation domain, it is worth mentioning the works [55,56] of Roupert et al. (2010) and reference therein, which in turn describe the construction of Total Differential (TD) three-phase data for the implementation of the exact global pressure formulation for the modeling of three-phase compressible flow in porous media. The difficulty in obtaining physically realistic TD three-phase relative permeabilities and capillary pressures data [56,55] has limited the use of the global pressure in numerical simulation codes. Nevertheless, when such data is available, this global formulation is preferred since it reduces the coupling between the pressure and saturation equations, compared to phase or weighted formulations [21,11]. In addition, such total differential three phase permeabilities formulation also simplifies the numerical analysis of the problem along with computational efficiency [55,56]. Therefore, it is combined in this work distinct methods for the resolution of the different type of equations that appears in the three-phase flow model, despite the presence or not of constitutive relationships of relative permeability, saturation and capillary pressure in three-phase transport in porous media which ensure hyperbolicity over the whole saturation domain [55,56,51,11,60–62].

For a wave to be truly defined as a "shock wave", a discontinuity must be the zero-diffusion limit of traveling wave solutions. For such solutions, diffusion balances the convergence of waves caused by hyperbolic nonlinearity [51,58,60,64]. Moreover, relevant to calculating nonclassical transitional shocks in two-phase [6] and three-phase [4,45] solutions is the correct modelling of the physical diffusive effects caused by capillary pressure mechanisms [10,23,55,56,4,6]. With gravity, the three-phase model yields elliptic regions for any combination of viscosities [12,64], and these regions occupy a significant fraction of the saturation space. Jackson and Blunt in [42] demonstrate that, even when capillary forces are small relative to viscous forces, they have a significant effect on solutions [64] for a realizable model of a porous medium. As a consequence that capillary pressure should be included explicitly in three-phase numerical simulators to obtain stable solutions which reproduce the correct sequence of saturation changes in the interior of the phase space. In addition, we remark that nonclassical waves have been identified mathematically for a number of other physical problems [65,57,59,13,14,17,42].

The fractional-step time-marching method discussed in this manuscript combines a conservative central scheme to handle a system of nonlinear conservation laws modelling the convective transport of the fluid phases with locally conservative mixed finite elements for the associated parabolic and elliptic problems. It has been shown in [2,3,5,45] that this technique can be effective in capturing the correct nonclassical fronts in one spatial dimension as well as in multidimensional heterogeneous three-phase gravity-free flows [4].

Similar ideas involving alternative fractional-step procedures as discussed in this work have been presented in [46–49]. But the work in [46,47] are based in a front tracking approach, which in turn relies heavily on a Riemann solver and the method of polygonal approximations of Dafermos [24]. Furthermore, in [49,48] the authors have been shown that the using semidiscrite central-upwind schemes may fail to converge to the unique entropy solution of nonconvex conservation laws, and thus may fail to recover the Kruzhkov solution [23,6]. On the other hand, the Strang splitting [63], as used in [7] is not an alternative since the hyperbolic nature of the three-phase differential flow system is not completely understood [40,58,51,60,11]. Indeed, the splitting approach proposed in [7] is based on the solution of a parabolic problem via a discretization of the formula for the exact kernel solution of a (linear) heat equation with constant coefficients as opposed in this work.