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Positive almost periodic solutions for shunting inhibitory cellular neural networks with time-varying delays

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Abstract

This paper is concerned with the existence and exponential stability of positive almost periodic solutions of shunting inhibitory cellular neural networks (SICNNs) with time-varying delays arising from the description of the states of neurons in delayed neural networks in a time-varying situation. By applying Lyapunov functional method and differential inequality techniques, new sufficient conditions ensuring the existence and exponential stability of positive almost periodic solutions for SICNNs are established.

MR (2000) subject classification: 34C25; 34K13; 34K25

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1. Introduction

Consider shunting inhibitory cellular neural networks (SICNNs) with time-varying delays described by

$$x'_{ij}(t) = -a_{ij}x_{ij}(t) - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl} f(x_{kl}(t-\tau(t)))x_{ij}(t) + L_{ij}(t), \tag{1.1}$$

where $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, C_{ij} denotes the cell at the (i, j) position of the lattice, the r-neighborhood $N_r(i, j)$ of C_{ij} is given by

$$N_r(i, j) = \{C_{kl} : \max(|k-i|, |l-j|) \le r, 1 \le k \le m, 1 \le l \le n\}.$$

x_{ij} acts as the activity of the cell C_{ij}, $L_{ij}(t)$ the external input to C_{ij}, the constant $a_{ij} > 0$ represents the passive decay rate of the cell activity, $C_{ij}^{kl} \ge 0$ the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij}, and the activity function $f(\cdot)$ is a continuous function representing the output or firing rate of the cell C_{kl}, and $\tau(t) \ge 0$ corresponds to the transmission delay.

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Since Bouzerdoum and Pinter in [1–3] described SICNNs as a class of new cellular neural networks(CNNs), SICNNs have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. The applicability and efficiency of such networks hinge upon their dynamics, and therefore the analysis of dynamic behaviors of the networks is a first and necessary step for any practical design and application of the networks. Recently, considerable effort has been devoted to study dynamic behaviors, in particular, the existence and stability of the equilibrium point, periodic and almost periodic solutions of SICNNs with time-varying delays and continuously distributed delays in the literature (see, e.g., [4,5,8–10,14] and the references therein). As a result, a set of very generic, in-depth, criteria for the stability has been obtained for SICNNs. In contrast, however, very few results are available on a generic, in-depth, existence and exponential stability of positive almost periodic solutions for SICNNs (1.1). On the other hand, the existence and stability of positive almost periodic plays a key role in characterizing the behavior of dynamical system (see [12,13,15]). Thus, it is necessary to investigate the existence and stability of positive almost periodic solutions of SICNNs (1.1).

The purpose of this paper is to present sufficient conditions of existence and exponential stability of positive almost periodic solutions of system (1.1). Our approach is based on basic analysis method in [11]. With such an approach, we establish new sufficient conditions ensuring the existence, uniqueness and exponential stability of positive almost periodic solutions of system (1.1). Our results not only can be applied directly to many concrete examples of SICNNs, but also extend, to a large extent, the results in [1–5,8–15]. Moreover, an example is provided to illustrate the effectiveness of our results.

Throughout this paper, assume that $\tau(t) : R \to R$ is an almost periodic function, and $0 \le \tau(t) \le \bar{\tau}$, where $\bar{\tau} \ge 0$ is a constant. For $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, assume that $L_{ij} : R \to [0, +\infty)$ are almost periodic functions. From the theory of almost periodic functions in [6,7], it follows that for any $\epsilon > 0$, $\exists l > 0$ such that for any interval with length l, there exists a number δ in this interval, and

$$|L_{ij}(t+\delta) - L_{ij}(t)| < \epsilon, \qquad |\tau(t+\delta) - \tau(t)| < \epsilon, \tag{1.2}$$

for all $t \in R$. Suppose that there exist constants $\underline{L}_{ij}$ and L_{ij}^{+} such that

$$0 < \underline{L}_{ij} = \inf_{t \in R} L_{ij}(t), \qquad L_{ij}^{+} > \sup_{t \in R} L_{ij}(t). \tag{1.3}$$

Throughout this paper, we set

$$\{x_{ij}(t)\} = (x_{11}(t), \dots, x_{1n}(t), \dots, x_{i1}(t), \dots, x_{in}(t), \dots, x_{m1}(t), \dots, x_{mn}(t)) \in R^{m \times n}.$$

For $\forall x(t) = \{x_{ij}(t)\} \in R^{m\times n}$, we define the norm $\|x(t)\| = \max\limits_{(i,j)}\{|x_{ij}(t)|\}$.

We also assume that the following conditions (T_1) and (T_2) hold:

(T_1) $f : R \to R$ is a non-increasing function on $[0, +\infty)$, and there exist constants M_f and μ_f such that

$$f(0) = 0, |f(u) - f(v)| \le \mu_f |u - v|, |f(u)| \le M_f, \quad \text{for all } u, v \in R.$$

(T_2) There exist constants $\delta_{ij} > 0$, $\eta > 0$ and $\lambda > 0$, $ij = 11, 12, \dots, 1n, \dots, m1, m2, \dots, mn$, such that

$$\delta_{ij} = a_{ij} - \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl} M_f, \qquad (\lambda - a_{ij}) + \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl} M_f + \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl} \mu_f \frac{L_{ij}^{+}}{\delta_{ij}} e^{\lambda \bar{\tau}} < -\eta < 0. \tag{1.4}$$

The initial conditions associated with system (1.1) are of the form:

$$x_{ij}(s) = \varphi_{ij}(s), \qquad s \in [-\bar{\tau}, 0], i = 1, 2, \dots, m, j = 1, 2, \dots, n, \tag{1.5}$$

where $\varphi_{ij}(\cdot)$ denotes real-valued continuous function defined on $[-\bar{\tau}, 0]$.

Definition 1.1. (see [6,7]) Let $u(t) : R \to R^{m\times n}$ be continuous in t. $u(t)$ is said to be almost periodic on R if, for any $\varepsilon > 0$, the set $T(u, \varepsilon) = \{\delta : \|u(t+\delta) - u(t)\| < \varepsilon, \forall t \in R\}$ is relatively dense, i.e., for any $\varepsilon > 0$, $\exists l > 0$ such that for any interval with length l, there exists a number δ in this interval, and $\|u(t+\delta) - u(t)\| < \varepsilon$ for all $t \in R$.