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On elliptical model for forest fire spread modeling and simulation

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Abstract

A new approach for the derivation of elliptical model of steady-state forest fire spread in time based on classical envelope theory is described. The derivation of this model known in the literature is based on the introduction of a special transform of co-ordinate system which allows to utilize geometrical properties of points lying on common tangent line of two circles. However, the use of this transform means the necessity to apply some specific assumptions onto the model. The proposed new procedure allows us to derive the model without the use of this transform. The new approach enables to better explain the internal coherence of studied problem, assumptions and limitations of the model, as well as to suggest its further generalizations.

Keywords: Fire behaviour modeling; Local elliptical fire spread; Huygens' principle; Envelope theory; Forest fire simulation

MSC: 93A30; 81T80; 68U35

1. Introduction

Mathematical modeling of forest fire spread in time plays a key role in existing decision support systems developed for planning, management and coordination of fire fighting activities and fire prevention. Forest fires belong to destructive natural phenomena which have not been sufficiently described because of their complexity, still inadequate knowledge about the processes determining behaviour of the phenomenon, and huge amount of required data, as well as of serious difficulties with their extraction and gathering. Every year, forest fires cause a large damage of vegetation, property, eco-systems and environment, but they also threaten people's lives and block significant human resources. Advances in computers and information technologies stimulate the development of various useful program systems for fire fighting. In particular, fire behaviour predicting systems are capable to simulate the growth of a forest fire front after the fire detection. They describe not only the spatial and temporal behaviour of forest fires (fire spread rate and direction), but can quantify and often even display various fire characteristics (e.g. fire intensity, flame length, etc.), which can be used for the fire effects evaluation. They can be directly used for the specific purposes of fire management or can be included in more complex decision support systems [6]. In fire prevention and planning, they can be used for simulation of various fire scenarios under different conditions in a certain region to test the fire management response for the fire event and the effectiveness of different types of suppression strategies and tactics, taking into account the existing fire fighting infrastructure and specific conditions that affect the fire fighting (e.g. location of water sources,

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road network, etc.). During the post-suppression stage, the systems are used for reconstruction of past forest fire events to better understand the circumstances that lead to human incidents due to the fire fighting (see e.g. [28–30,15]). The fire behaviour predicting systems are also useful for operational purposes and training. The current generally little use of decision support systems for the forest fire management in the European Union has become an urgent challenge for scientists, researchers and developers, as well as for responsible state and regional authorities [32].

The knowledge of mathematical model for description of the spread of a fire front (burning area boundary) in time under variable topographic, meteorological and fuel conditions is significant not only for better understanding the model itself, but also for its effective implementation, correct use, and proper interpretation of results obtained during fire spread simulations under real conditions. The fire front is usually represented by a planar or spatial curve. The fire spread in time is described either by a set of sophisticated rules for a local fire spread obtained by averaging the typical fire behaviour observed during wildland and/or experimental fires, or by a system of differential equations representing the relation between various physical quantities influencing the fire behaviour. From different points of view, the forest fire spread models are usually classified into two, and three main classes: *stochastic* and *deterministic* models [24,8,16], and *empirical*, *semi-empirical* and *physical* models [23,10,5,31,21,19]. Stochastic models are based on the observation of experimental and wildland fires from which the fire spread rate is related to relevant burning parameters (fuel type, fuel moisture, wind, etc.) in a statistical way. They accumulate knowledge obtained from laboratory and outdoor experimental fires to predict a more probable fire behaviour from average conditions. However, the obtained empirical relations depend strongly on very specific conditions from which the statistical analysis is performed. Such models are also referred to as *empirical* models. Deterministic models are usually divided to semi-empirical and physical models. *Semi-empirical* models are based on a global energy balance expression and on the assumption that the energy, which is transferred to the unburned fuel, is proportional to the energy released by the combustion of the fuel. Many useful relations of the model were obtained by fitting to extensive laboratory fire experiments. The simplicity of this approach allows to develop effective operational tools for forest fire simulation under real conditions such as e.g. BEHAVE and FARSITE. *Physical* models take into account one or several processes of energy transfer from the burning zone to the unburned fuel. In general, physical models lead to differential equation systems which require sophisticated and time-consuming numerical calculations and advanced high-performance computing environments [20]. The simulation of real more extensive forest fires, realized on currently available computer equipment and applicable for operational purposes, is generally restricted for semi-empirical and empirical models.

A huge work has been done on the analysis and description of *local fire spread* on a flat ground in uniform continuous non-spotting fuels involving constant wind velocity, moisture content and slope. The most often used analytical approximation of the fire shape is that of an ellipse [14,27]. This model is known in the related literature as a *local elliptical model* of the fire spread. Other approximations known from the literature are “teardrops” [27], “ovoids” [22] and “double ellipses” [1,3,26]. This paper is motivated by studies on the application of Huygens’ principle of wave propagation on the problem of description of *global steady-state forest fire spread* [4,25] based on the assumption of the local elliptical fire spread. Each point on a starting fire front at a given time t can be considered as an ignition point of a small local fire which causes burning out of the area of elliptical shape at time $t + \mathrm{d}t$. Assuming that each such ellipse is defined by burning conditions at its generating ignition point, the resulting fire front at time $t + \mathrm{d}t$ can be defined by the envelope of all the ellipses (*Huygens’ principle* for fire spread [4]). The *system of differential equations* for description of the global steady-state forest fire spread in time was derived analytically by Richards [25]. His approach has become the basis for several successful software systems for *simulation of forest fire spread under real conditions*. The system of differential equations was obtained by a special transform (rotation and comprimation of axes of co-ordinate system) which allowed to transform the ellipses into circles and then to utilize some specific geometrical properties of points lying on a common tangent line of two circles. The use of this transform, which corresponds to the introduction of specific additional assumptions on semi-axes of the corresponding ellipses, was inevitable for this approach and determined the final form of the equations derived [12]. In this paper, we show that it is possible to derive the investigated fire spread model without the use of the mentioned transform applying the classical theory of envelopes of curves sets.

A number of assumptions are critical to modeling the fire growth. Some of these assumptions are probably not strictly met by current modeling methods. The discussion on the assumptions and limitations of semi-empirical models and specifically of concrete implementations of the Richards’ method is well known in the related literature (for more information see e.g. [25,5,11,15,9]). The discussion includes the questions such as the independence of the fire spread of the shape or length of the fire front; the independence of the fire spread and intensity at a given point of the fire