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Minimisation of the risk of trampling in a crowd

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Abstract

Over the past decade, there have been many crowd related tragedies. To help avoid such situations a strategy is developed here to improve the safety of pedestrians in densely populated situations. The results of simulations performed on two cases of accidents involving trampling, which occur when pedestrians are moving, illustrate the ability of this modelling strategy for minimising predicted crowding risks in such situations. This study demonstrates that effective crowd control may be achieved either by adjusting the size of the crowd or the complexity of the environment in which pedestrians walk, which effectively influences their speed.

Keywords: Pedestrian safety; Crowd motion; Trampling

1. Introduction

Human crowds have been studied for more than three decades. However, studies of the flows of large crowds of pedestrians associated with special events are limited when compared with those of pedestrian flows in normal walking environment. In large crowds, there is a potential for injury and even loss of life resulting from the dynamics of the crowd's behaviour. Given the increasing number of large-scale sporting events, religious gatherings and rock concerts with time, the issue of crowd safety is of growing importance. In order to minimise risks in densely populated situations, a proper understanding of motion and inherent dangers of large crowds of pedestrians is essential.

It has been argued that the behaviour of crowds is unlikely to be able to be predicted easily because crowds are irrational and erratic. To the contrary, the modern sociological understanding is that an unorchestrated crowd behaves rationally as described by [5]. Also of importance here, Aristotle and much more recent studies have noticed that actions are generally motivated by goals [7]. Human behaviour is seen as goal-directed, i.e. people develop goal hierarchies, which influence their decisions as time evolves. Therefore, for large crowds of pedestrians attending events, such as sporting events or religious gatherings, the behaviour of such crowds can be described as rational and goal-directed because the members of the crowds have clear knowledge of their goals and where they lie. The motion of a crowd can therefore be modelled using rational principles.

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A previous study [4] has investigated the mechanics of flows of large crowds of pedestrians in dangerous situations, which could result in loss of life due to either trampling of pedestrians in a crowd or crushing of pedestrians, and has illustrated and explained the development of such situations by developing models using a continuum theory for crowd motion at both high and very high densities of pedestrians. In the present study, this theory is extended to develop a strategy, as presented in Section 2, for minimising the risks of trampling in a crowd. Section 3 considers the influence of the complexity of environment on the walking speeds of pedestrians, followed by the conclusions drawn from this study.

2. Minimisation of crowding risks

In [3] equations of motion of a continuum crowd of pedestrians are derived as

$$-\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho f^2(\rho)\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\rho f^2(\rho)\frac{\partial \phi}{\partial y}\right)=0 \tag{2.1}$$

and

$$f(\rho)=\frac{1}{\sqrt{(\partial\varphi/\partial x)^2(\partial\varphi/\partial y)^2}} \tag{2.2}$$

Here ρ represents the density of the crowd, which is measured by the expected number of pedestrians found per unit area, t represents time, $f(\rho)$ corresponds to the desired walking speed of pedestrians in the style of [1] and [6] (both originally for vehicular flow) and is a function of density ρ, ϕ is the remaining travel time (called the velocity potential) defined by $u=-f^2(\rho)\,\partial\phi/\partial x$ and $v=-f^2(\rho)\,\partial\phi/\partial y$, in which (u, v) is the local velocity of the crowd in the (x, y) coordinates.

These equations correspond to those first obtained by [2]. Many forms of $f(\rho)$ have been proposed in the literature on the macroscopic characteristics of pedestrian traffic flow. For our purpose, $f(\rho)$ can be approximated by

$$f(\rho)=A\,\mathrm{e}^{-\rho/\rho_{\text{crit}}} \tag{2.3}$$

where ρ_{crit} represents the density at which the pedestrian flow, $\rho f(\rho)$, is maximum, with typical value of 3 ped/m^2, and A is the free walking speed when the density of pedestrians approximates zero. A typical value for A is 1.4 m/s.

A previous study [4] has shown that the above theory was able to identify the dangerous locations within a crowd and explain the development of trampling and crushing accidents. A full derivation of these equations can be found in [3].

The dangerous situations considered in the present study, the crowd density of interest is high and so the flow of pedestrians considered here is subcritical, that is slow-moving and high-density flow. In such case, spatial variations in ρ are small compared to those in ϕ, that is

$$\frac{\partial^2\phi/\partial x^2}{\partial\phi/\partial x}\rangle\rangle\frac{\partial\rho f^2/\partial x}{\rho f^2}$$

As a result, the governing equation of crowd motion, (2.1), simplifies to Laplace's equation:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \tag{2.4}$$

The density of pedestrians is an important isotropic quantity that appears to determine the probability of a fatal crowding accident occurring [4]. The literature on pedestrian motion and the study by [4] on modelling the development of crowd related accidents suggest that the probability, P_{r}, of such an accident occurring in area $\delta x\delta y$ over some time interval δt, given that an accident occurred somewhere in the region in time interval δt, may be approximated by

$$P_{\mathrm{r}}=\frac{\mathrm{e}^{\beta\rho}\,\delta x\,\delta y}{\iint \mathrm{e}^{\beta\rho}\,\mathrm{d}x\,\mathrm{d}y} \tag{2.5}$$

where ρ is the density of pedestrians, the integral is over the total area where accidents may occur (the domain of the solution) and β is a parameter related to ρ_{a}, the density at which a crowd related accident is most likely to occur, as