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Lattice Boltzmann simulation of natural convection in porous media

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Abstract

This paper confirms the reliability and the computational efficiency of the lattice Boltzmann method in simulating natural convection in porous media at the representative elementary volume scale. The influence of porous media is considered by introducing the porosity to the equilibrium distribution function and by adding a force term to the evolution equation. The temperature field is simulated by a simplified thermal energy distribution function which neglects the compression work done by the pressure and the viscous heat dissipation. A comprehensive parametric study of natural convective flows is carried out for various values of Rayleigh number, of Darcy number, and of porosity. The comparison of solutions between the present model and earlier studies shows good quantitative agreement for the whole range of Darcy and Rayleigh numbers.

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Keywords: Lattice Boltzmann method; Natural convection; Porous media

1. Introduction

A lot of early researchers on natural convection in porous media used the Darcy's equation. For high velocity, many experimental data do not agree with the theoretical prediction based on Darcy's law [2]. Two notable modifications of the Darcy's equation are proposed: one is Forchheimer's equation [5] that takes consideration of non-linear drag effect due to the solid matrix, and the other is Brinkman's equation [1] that includes the viscous stresses introduced by the solid boundary. For the case in which the Reynolds number or the Darcy number is large, the non-linear drag must be considered. The convective heat transfer is mostly a boundary phenomenon; Brinkman's modification for the Darcy's equation is significant for the energy transport process. Although it is not easy to combine these two equations, the Brinkman–Forchheimer equation which includes the viscous and inertial terms was derived by the local volume averaging technique [8]. Many researchers calculated the Brinkman–Forchheimer equation using conventional numerical methods and demonstrated the equation can appropriately predict the heat transfer and fluid dynamics in the non-Darcy regime [7,9]. The lattice Boltzmann method (LBM) has been successfully applied to study of the isothermal flows in porous media not only at the pore scale [12] but also at the representative elementary volume (REV)

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scale [6]. We will confirm the reliability of the LBM in simulating natural convection in the porous media with the Brinkman–Forchheimer equation.

2. The lattice Boltzmann model

The continuity equation, the Brinkman–Forchheimer equation, and the energy equation are written as:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\left(\frac{\mathbf{u}}{\varepsilon}\right) = -\frac{1}{\rho}\nabla(\varepsilon p) + \nu_{\mathrm{e}}\nabla^2\mathbf{u} + \mathbf{F}, \tag{2}$$

$$\partial_t(\rho e) + \nabla \cdot (\rho \mathbf{u} e) = \chi \nabla^2(\rho e), \tag{3}$$

where ε is the porosity of the medium, ν_e the effective viscosity, and χ is the thermal diffusivity. The total body force $\mathbf{F}$ encompasses the viscous diffusion, the inertia due to the presence of a porous medium, and an external force. With the widely used Ergun's relation [4], the body force can be written as

$$\mathbf{F} = -\frac{\varepsilon\nu}{K}\mathbf{u} - \frac{1.75}{\sqrt{150\varepsilon K}}|\mathbf{u}|\mathbf{u} + \varepsilon\mathbf{G}, \tag{4}$$

where ν is the kinematic viscosity, K is the permeability, and $\mathbf{G}$ is the gravity. The thermal energy distribution LB model solves the following kinetic equations for the distribution functions f_i and g_i [10],

$$f_i(\mathbf{x} + \mathbf{e}_i\delta_t, t + \delta_t) - f_i(\mathbf{x}, t) = -\frac{f_i(\mathbf{x}, t) - f_i^{\mathrm{eq}}(\mathbf{x}, t)}{\tau_v} + \delta_t F_i, \tag{5}$$

$$g_i(\mathbf{x} + \mathbf{e}_i\delta_t, t + \delta_t) - g_i(\mathbf{x}, t) = -\frac{g_i(\mathbf{x}, t) - g_i^{\mathrm{eq}}(\mathbf{x}, t)}{\tau_c}. \tag{6}$$

Equation (5) recovers the continuity and the momentum Eqs. (1) and (2). Equation (6) describes the evolution of the internal energy and leads to Eq. (3). The macroscopic quantities, fluid density and internal energy, are defined as $\rho = \sum_i f_i$ and as $e = \sum_i g_i/\rho$, respectively. The fluid velocity $\mathbf{u}$ is calculated using a temporal velocity $\mathbf{v}$ to consider the effects of porous media.

$$\mathbf{u} = \frac{\mathbf{v}}{c_0 + \sqrt{c_0^2 + c_1|\mathbf{v}|}}, \tag{7}$$

where $\mathbf{v} = \sum_i \mathbf{e}_i f_i/\rho + \frac{\delta_t}{2}\varepsilon\mathbf{G}$, $c_0 = \frac{1}{2}\left(1 + \varepsilon\frac{\delta_t}{2}\frac{\nu}{K}\right)$, and $c_1 = \varepsilon\frac{\delta_t}{2}\frac{1.75}{\sqrt{150\varepsilon^3 K}}$. The equilibrium distribution function f_i^{eq} for the $D2Q9$ model is given by

$$f_i^{\mathrm{eq}} = \omega_i\rho\left[1 + \frac{3\mathbf{e}_i \cdot \mathbf{u}}{c^2} + \frac{9(\mathbf{e}_i \cdot \mathbf{u})^2}{2\varepsilon c^4} - \frac{3\mathbf{u}^2}{2\varepsilon c^2}\right], \tag{8}$$

where ω_i is the weight, and c is the lattice spacing. For the $D2Q9$ model, the discrete velocities are defined by $\mathbf{e}_0 = \mathbf{0}$, $\mathbf{e}_i = c(\cos((i-1)\pi/2), \sin((i-1)\pi/2))$ for $i = 1$–4, and $\mathbf{e}_i = \sqrt{2}c(\cos((i-5)\pi/2 + \pi/4), \sin((i-5)\pi/2 + \pi/4))$ for $i = 5$–8. The weights are given by $\omega_0 = \frac{4}{9}$, $\omega_i = \frac{1}{9}$ for $i = 1$–4, and $\omega_i = \frac{1}{36}$ for $i = 5$–8. Similarly the equilibrium distribution functions for the thermal energy distribution g_i^{eq} can be written as

$$g_0^{\mathrm{eq}} = -\frac{2\rho e}{3}\frac{\mathbf{u}^2}{c^2}, \tag{9}$$

$$g_i^{\mathrm{eq}} = \frac{\rho e}{9}\left[\frac{3}{2} + \frac{3}{2}\frac{\mathbf{e}_i \cdot \mathbf{u}}{c^2} + \frac{9}{2}\frac{(\mathbf{e}_i \cdot \mathbf{u})^2}{c^4} - \frac{3}{2}\frac{\mathbf{u}^2}{c^2}\right] \quad (i = 1, 2, 3, 4), \tag{10}$$

$$g_i^{\mathrm{eq}} = \frac{\rho e}{36}\left[3 + \frac{6\mathbf{e}_i \cdot \mathbf{u}}{c^2} + \frac{9}{2}\frac{(\mathbf{e}_i \cdot \mathbf{u})^2}{c^4} - \frac{3}{2}\frac{\mathbf{u}^2}{c^2}\right] \quad (i = 5, 6, 7, 8). \tag{11}$$